WHY IS PIGOU SOMETIMES WRONG? UNDERSTANDING HOW DISTORTIONARY TAXATION CAN CAUSE PUBLIC SPENDING TO EXCEED THE EFFICIENT LEVEL

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ABSTRACT

When a public good is financed by a proportional tax, the price distortion increases the marginal cost of the public good above its resource cost. Pigou (1928) conjectured that the higher cost lowers the second-best public good level below the first-best level. I explain how the price distortion is likely to also raise the marginal benefit of the public good, so that Pigou's reasoning is incomplete. The second-best public good level exceeds the first-best level when the price distortion increases the marginal benefit more than the marginal cost.

Key words: proportional tax, second-best, public service level.

1. INTRODUCTION

With lump-sum taxation, the first-best public good level is characterized by the "Samuelson Rule": expenditure on the public good should be increased until the marginal benefit equals the marginal resource cost.[1] The use of lump-sum taxation to finance a public good is of course unrealistic, but it is a useful benchmark against which to compare other tax structures. If the lump-sum tax is replaced by a proportional tax, there is a price distortion - the consumer price exceeds the marginal cost of the taxed good - which gives rise to a welfare cost; this welfare cost is henceforth termed the “distortion cost”. Pigou (1928, p. 53) recognized that the distortion cost should be added to the resource cost to obtain the “full” marginal cost of the public good, and conjectured that the increased cost would cause the second-best public good level to fall below the first-best level. Atkinson and Stern (1974), King (1986) and Wilson (1991a) confirm that the second-best public good level is indeed less than the first-best level when the household's utility function has either Cobb-Douglas or CES form. However, Pigou’s conjecture is not generally correct: de Bartolome (1998), Gaube (2000) and Gronberg and Liu (2001) provide examples in which the second-best public good level exceeds the first-best level.

Pigou's intuition hinges on the way the proportional tax increases the marginal cost of the public good. What is missing in his analysis - and what is discussed in this paper - is the effect of the distortion on the marginal benefit of the public good.[2] The marginal benefit of the public good is measured as the quantity of numeraire which households are willing to give up to obtain an additional unit of the public good. By changing the bundle of commodities consumed, the distortion changes the household’s willingness to give up the numeraire. In particular, using a proportional tax instead of a lump-sum tax causes the household to substitute into - or increase its consumption of - the untaxed numeraire. This increased consumption is likely to lower the marginal utility of the numeraire. With the numeraire giving less marginal utility, the household is more willing to give it up to gain the public service, or the marginal benefit of the public good increases. If the marginal benefit increases more than the marginal cost, the net effect of the distortion is to increase the second-best public good level above the first-best level.

That the price distortion affects the marginal benefit of the public good as well as its marginal cost enables me to provide some intuition for the result of Gaube (2000) that, if the second-best public good level exceeds the first-best level, the numeraire must be a Marshallian complement with the taxed good. It also enables me to show that, if the second-best public good level exceeds the first-best level, the taxed commodity must be normal. And it provides intuition for Wilson's (1991b) result. In Wilson's model, there are dissimilar households so that the planner wants to redistribute resources between households in addition to providing the public good. Wilson shows that the second-best public good level may exceed the first-best level and attributes this to the gain of shifting resources from the distorted private sector to the public sector. This paper provides an alternative explanation: redistribution is achieved by using a

2. THE MODELS

The population is comprised of a large number of households, each of which has identical tastes and income. A representative household consumes leisure ℓ, a commodity x, and the public service[3] z. The utility achieved by the household is assumed to be additively separable between private goods and the public service, or is

$$U(\ell, \quad .$$

2.1 First-best Allocation.

The benchmark analysis is the first-best case in which the public service is financed by a lump-sum tax imposed at the first-stage. At the second stage, the household takes the tax .

The left-hand side of Equation (3) is the marginal benefit of the public service when financed by a lump-sum tax and is denoted as $MB^F(z)$; the marginal benefit is measured as the amount of numeraire the household is willing to give up to gain an extra unit of z. The right-hand side of Equation (3) is the marginal cost of the public service when financed by a lump-sum tax and is denoted $MC^F(z)$. It is shown in Appendix A that $MB^F(z)$ is a decreasing function of z.

Figure 1 (shown in the next section) shows both the first-best and second-best allocations. Figure 1(a) considers the allocation of ℓ and x, and Figure 1(b) considers the determination of z. In this section we are considering the first-best allocation. In Figure 1(a), AB is the household's budget line if $z = 0$. If $z > 0$, the lump-sum tax shifts the household's budget line to CD and the household's allocation (ℓ^F

The household's choice of x conditional on q, $x^S(q)$, is implicitly defined by the first-order condition[6]

$$\frac{U_x(}{} \qquad (4)$$

In Figure 1(a), the commodity tax raises the consumer price of *x* above *p*, or the household's budget line swivels from *AB* to *AE*. If the same level of public service is provided as in the lump-sum case, the government must choose the tax rate so that the same resources *kz* are collected from the household or so that the household's budget line *AE* induces the household to choose an allocation on *CD*: facing the budget line *AE* , the household achieves the indifference curve *I'* at *S*. $x^S(q(z))$ is the consumption of *x* at *S*. The commodity tax causes a consumption distortion - the household consumes less commodity *x* and more numeraire than if a lump-sum tax is used. This is formalized in the Lemma below.

LEMMA 1: $R^S(q(z)) > R^F(z)$.

PROOF: $U_x(H -$

At the first stage, the government chooses the public service level (and the implied t

rate) as

$$\max_{z} \; U(H - q(z)\, x^S(q(z)),\; x^S(q(z))) \; + \; G(z).$$

Differentiating to obtain the first-order condition,

$$U_\ell \left(-q \frac{dx^S}{dq}\frac{dq}{dz} - x^S \frac{dq}{dz} \right) \; + \; U_x \frac{dx^S}{dq}\frac{dq}{dz} \; + \; G$$

The left-hand side of Equation (9) is denoted $MB^S(z)$, the marginal benefit of the public service when financed by the commodity tax. The right-hand side, being the marginal resource cost multiplied by the marginal cost of funds[7] , is the marginal cost of the public service when financed by the commodity tax, denoted $MC^S(z)$. The assumption that $\gamma > 0$ implies that the right-hand side of the Equation (9) exceeds k or that, in Figure 1(b), $MC^S(z)$ lies strictly above $MC^F(z)$ for $z > 0$.

The second-best efficient level z^S occurs at the intersection G' of $MB^S(z)$ and $MC^S(z)$. Whether $z^F > z^S$ (as Pigou conjectured) or $z^S > z^F$ depends on whether and by how much $MB^S(z)$ lies above $MB^F(z)$. This is the topic to which I now turn.

3. DISTORTION AND PUBLIC SERVICE LEVEL

3.1 The shift in the marginal benefit curve

The focus of this paper is how the distortion associated with the use of the commodity tax shifts the marginal benefit schedule of the public service. Lemma 1 establishes that, at any value of z, using the proportional tax causes the household to consume more numeraire and less of the taxed good than when a lump-sum tax is used, or to move along CD from F to S in Figure 1. This affects the marginal utility U_ℓ. Each small movement along CD away from F and towards S affects the marginal benefit schedule at a pre-determined z as

$$\frac{d}{d\ell}\,\frac{G_z(z)}{U_\ell\left(\ell\,,\,\dfrac{H-\ell-kz}{p}\right)} \;=\; -\,\frac{G_z}{U_\ell^2}\left[U_{\ell\ell}-\frac{U_{\ell x}}{p}\right]. \tag{10}$$

If x is inferior, $- Q U_{RR} + U_{Rx} < 0$. With $Q \$ p$, this is sufficient to ensure that $- p U_{RR} + U_{Rx} < 0$ at all points on FS, and the marginal benefit schedule shifts down. In this case, $MB^S(z)$ lies below $MB^F(z)$

Summarizing, x being normal favors but does not ensure that moving to the second-best from the first-best shifts up the marginal benefit schedule. However, x being inferior does ensure

Assumption 2 ensures that the distortion shifts up the marginal benefit schedule of the public service.

3.4 Substitutability/Complementarity

Several results in the literature depend on whether R and x are substitutes or complements. E.g. Gaube (2000, Proposition 1) shows that, if R and x are normal and if R is a Marshallian substitute for x (i.e. $\partial R/\partial q > 0$), then $z^F > z^S$. Put differently, if R and x are normal goods, a necessary (but not sufficient) condition for $z^S > z^F$ is that R is a Marshallian complement for x. The importance of complementarity can be understood from Equation (10). Loosely, R and x are complements when R gives more utility if it is used in the presence of x, or complementarity is favored if $U_{Rx} > 0$.[9] As noted in Section 3.1, $U_{Rx} > 0$ implies that, if a commodity tax replaces the lump-sum tax, the implied decrease in x lowers the marginal utility of R, increasing the upward shift in $MB^S(z)$. This favors z^S exceeding z^F.

3.5 Wilson's (1991b) model

Wilson (1991b) provides an example in which the second-best public service exceeds the first-best. His model has many dissimilar households, and the government must determine the extent of redistribution and the public service level. In the first-best, redistribution and the financing of the public service are achieved using individualized lump-sum taxes. In the second-best, a commodity tax is used for redistributional concerns and any additional tax revenue is collected using a uniform lump-sum tax. Although the commodity tax, the lump-sum tax and the public service level are determined simultaneously, the second-best analysis is done with the commodity tax being pre-determined (at its optimal level) so that any marginal change in the public service level is financed by changing the lump-sum tax. In this framework Wilson provides an example in which the second -best public service level exceeds the first-best level.

Using the framework of a representative household (in which there can be no redistributional motives), we can see the forces at work by supposing that in the first-best there is only a lump-sum tax but that in the second-best there is a pre-existing commodity tax rate $\bar{q}$ (potentially financing a lump-sum transfer) and that any shortfall in the tax-receipts required to finance the public service is made-up by a lump-sum tax. The first-best analsysis has already been described in Section 2.1 and I now turn to the second-best analysis.   At the second stage of

$$. \tag{12}$$

In Figure 2(a), *AB* is the economy's budget line if $z=0$ (and the price of x is its marginal resource cost p). If the public service is z, then kz must be withdrawn from the household or the budget line of the economy is *CD*. *F* is the first-best allocation (conditional on z). If the same level of the public service is provided but the consumer price of x is now $\bar{q}$, $\bar{q}>p$, and the lump-sum tax is T , $T<kz$, the consumer's budget line is *JK*. The consumer's choice *W* is where his indifference curve just touches his budget line and the government must chose the lump-sum tax *T* so that *W* lies on *CD*. Comparing *W* with *F*

(relative to the first-best): the logic of the previous sections suggests that this bundle distortion typically lowers the marginal benefit schedule, ceteris paribus favoring $z^S < z^F$.

Maintaining the structure of the previous sections, utility is considered to have the form:

$$U(\ell, x) - A(E) + G(z)$$

where E represents an aggregate negative externality associated with the consumption of x. If N is the number of households, $E = Nx$. In addition, $A_E > 0$. Using the economy's resource constraint $\ell + px + kz = H$ to substitute for ℓ, the first-best planner's problem is:

$$\max_{z} \max_{x} \quad U(H - px - kz, x) \ - \ A(Nx) \ + \ G(z).$$

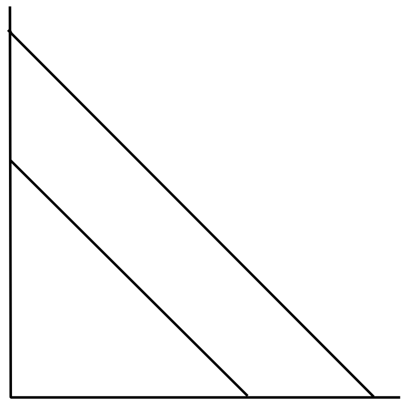
As in Section 2, write the solution to the second-stage as $x^f(z)$. It is defined by the first-order condition:

$$\frac{U_x}{U_\ell} = p + N\frac{A_E}{U_\ell} > p.$$

The slope of the household's indifference between ℓ and x is x's resource cost plus the external cost of consuming a marginal unit of x. At the first stage, the government chooses z such that

$$\frac{G_z(z)}{U_\ell(H - px^f(z) - kz, x^f(z))} = k \ . \tag{14}$$

The left-hand side of Equation (14) is the marginal benefit of the public service, $MB^f(z)$. The right-hand side is the marginal cost, $MC^f(z)$. In Figure 3(a), AB is the economy's resource constraint if $z = 0$. With a lump-sum tax kz, the resource constraint is CD. The negative externality raises the social price of x, so the first-best outcome (conditional on z) is the point F on CD where the slope of the indifference curve is $p + N A_E/U_R$.



1. "Marginal benefit" is used synonymously with "marginal rate of substitution" throughout the paper. Similarly, "first-best" and "second-best" are used synonymously with "first-best efficient" and "second-best efficient."

2. Ng (2000, p. 256) and Batina and Ihori (2005, p. 40) note that the marginal benefit curve is affected by a commodity tax. Chang (2000) is also a useful reference on this topic.

3. The assumption of a public service is made to simplify the presentation. The results apply if the government expenditure is on a public good.

4. If the public service is complementary with the taxed commodity, an increase in the public service interacts with the pre-existing tax structure to create additional tax revenue, lowering the marginal cost of funds (Diamond and Mirrrlees (1971)). Separability avoids this additional effect.

5. The chosen x depends on its price p and household income M, and is written in traditional notation as $x(p; M)$. I set $x^F(z) \;/\; x(p; H - kz)$.

6. The chosen x depends on the consumer price q and household income M, and is written in traditional notation as $x(q; M)$. I set $x^S(q) \;/\; x(q; H)$.

7. The marginal cost of funds measures the units of numeraire the household needs as compensation if one unit of additional tax revenue is raised using tax instrument q. Writing tax revenue as R, the tax rate as $q(R)$ and the expenditure function as $e(q, U)$,

$$MCF = \frac{\partial e}{\partial q}\frac{\partial q}{\partial R} .$$

Using Shephard's Lemma, $Me/Mq = x$. $R = (q-p)x$; differentiating with respect to R