Working Paper No. 12-02

# How Trade Restrictions Disperse: Policy Dynamics with Firm Selection

Soojae Moon
University of Colorado at Boulder

October 2012, Revised November 2012

Department of Economics

University of Colorado at Boulder
Boulder, Colorado 80309

# How Trade Restrictions Disperse: Policy Dynamics with Firm Selection

Soojae Moon

November 7, 2012

Abstract

cycles, and I nd that the result for both analyzed economies (home and foreign) are worse after the implementation of trade restrictions than would have otherwise been the case of the economic depression.

Figure 1: Yearly World Exports, 2000-2010 (OECD database)

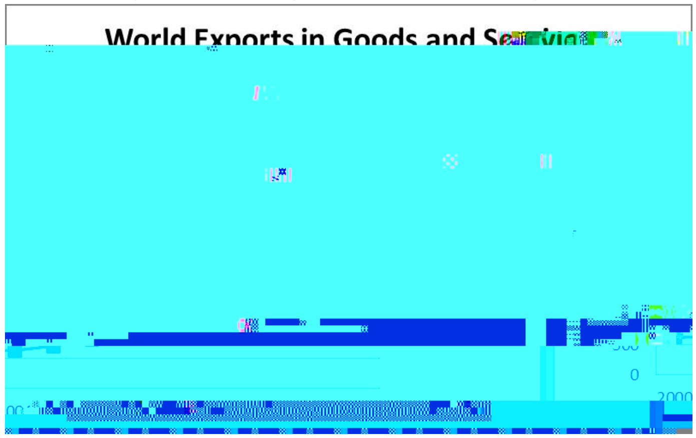
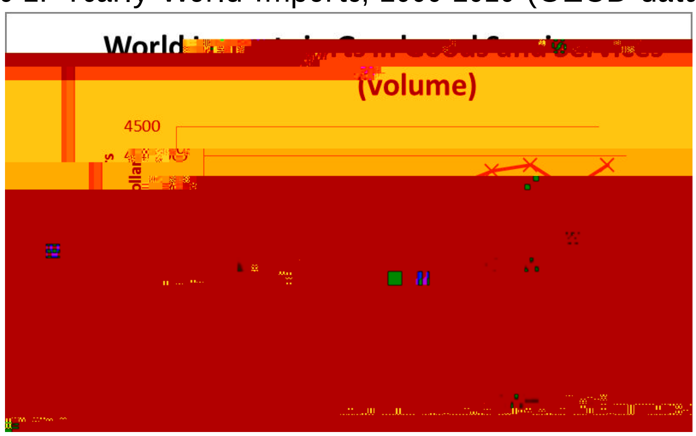


Figure 2: Yearly World Imports, 2000-2010 (OECD database)




Conventional international real business cycle (IRBC) models[4] assume international trade paradigms as exogenously given. An emerging class of IRBC models (New International out of the market or sell only in domestic market. To achieve the objective of this paper, the benchmark model is built based on this emerging class of trade micro-founded IRBC models that are suitable for analyzing the aggregate effects of change in trade policy such as tariffs and quotas. Ghironi & Melitz (2005) analyze precise endogenous Harrod-Balassa-Samuelson effect[5] using endogenous tradability with heterogenous firm-specific productivity, extending the Melitz (2003) model to embed it in dynamic and stochastic framework. However, they only analyze the long-run consequences. Alessandria & Choi (2007) study whether sunk costs of exporting matter along the business cycles. They conclude that entry costs only matter for the firm-level dynamics, but have little effect on aggregate fluctuations. They use endogenous labor and capital as inputs, but they do not consider the entry process and treat the fraction of exporters as constant. Bergin & Corsetti (2008) and Bilbiie, Ghironi & Melitz (2008) study monetary policy, incorporating firm entry and nominal price rigidities. They find that monetary shock has significant effects on firm entry. Bilbiie et al. (2008) document that profits are positively correlated and markups are negatively correlated with income in their model. These are features of the data that previous IRBC models had a hard time explaining.

I present a two-country, dynamic, stochastic, general equilibrium (DSGE) model with firm selection and variable adjustment of markup. As in Bergin & Glick (2007) and Ghironi & Melitz (2005), the model incorporates firms' entry and exit process along with firm heterogeneity. Firms know their productivity only after entry and the tradability of its good is endogenously determined. This endogenous tradability determine the firm's export condition where the least productive firms sell only in the domestic market, and the most productive firms sell in foreign markets. The model also incorporates a sunk entry cost and iceberg trade costs that affect the decisions of monopolistically competitive intermediate goods producers. Before entering the market, producers have to pay a fixed

[5] Harrod-Balassa-Samuelson (HBS) or Balassa-Samuelson (BS) effect is that wealthier economies have higher average prices relative to their trading partners. As a result, the terms of trade or exchange rate appreciate when there is a positive aggregate productivity shock in the home economy.

entry cost. Afterwards, they learn productivitiy, which is drawn from a Pareto distribution. Also, variable markups are introduced as a new avenue of `toughness' of rms' competition in a market such that competition will be tougher, rms charge lower markups, and aggregate productivity is higher. The variable adjustment of markups is generated from the non-homothetic preference of the nal goods technology taken from Melitz & Ottaviano (2008) and Ottaviano, Tabuchi & Thisse (2002). Melitz & Ottaviano (2008) derive the intra-industry reallocation e ects[6] and monopolistically competitive producers as in Melitz (2003), but add a new pro-competitive e ect of trade through lowering markup[7]. They use a non-homothetic quasilinear-quadratic function as a consumer's utility function that makes it hard to manage the general equilibrium model[8]. Therefore, I use household' utility function as in Ghironi & Melitz (2005), but instead use non-homothetic and non-constant elasticity of substitution aggregates in the nal goods production function. I assume that the nancial asset markets are incomplete to exist some degree of international risk sharing mechanisms[9], but not perfect.

There is a growing line of literature that uses non-constant elasticity of substitution to explain behavior of international relative prices and how the composition of aggregate income a ects trade patterns. Recently, several micro trade theory papers have incorporated non-homothetic preferences into their models. Foellmi, Hepenstrick & Zweimller (2011) explore the non-homothetic preferences into the new trade theory framework and compare its equilibrium outcomes with the case of standard homothetic preferences. Markusen (2010)

[6] Micro trade literature strongly approve these reallocation e ects of trade with heterogeneous rms. These e ects arise from rm selection of export status or trade liberalization. See Chaney (2008), Bernard, Jensen & Standarda Standar-dk751,

and Simonovska (2010) aggregate differentiated consumer goods using variable elasticity of substitution preferences and explain several existing trade puzzles. Goksel (2009) present a multi-country general equilibrium model of trade with non-homothetic preferences and find that differences in income with trading partner act as trading barriers. This approach is seen not only in micro-trade papers, but also in business cycle literatures. Ottaviano (2011) presents a business cycle model with a non-homothetic utility function that is defined over a continuum of horizontally differentiated products, exogenous labor, and endogenous capital. He argues that existing models overstate the role of heterogeneous firms and endogenous entry as a transmission of aggregate productivity shock because of asymmetric size effect of firms on aggregate fluctuations. Sakane (2011) studies the terms of trade dynamics, incorporating non-homothetic preference into the consumption index with endogenous labor supply. Using vector autoregression (VAR) and maximum forecast error variance identification, she analyzes the consequences of the US labor productivity shock on the terms of trade in different asset market assumptions. Rodriguez-Lopez (2011) studies exchange rate pass-through,[10] building a model with sticky wage, heterogeneous firms and endogenous markups. Davis & Huang (2010) incorporate endogenous markup into a model with nominal rigidities and investigate IRBC properties, but their model does not have entry and exit dynamics.

There is also much literature on gains from trade openings analyzing long-run equilibrium of models. Melitz (2007) proposes a dynamic model of firm-level adjustment to trade liberalization that captures the entry, exit, export, and innovation decisions of heterogeneous firms. They find that the timing and the speed of trade liberalization matters for firm-level productivity improvement and the entry decisons to the export market. Alessandria & Choi (2011) estimate the effect of reducing tariffs on welfare, trade, and export participation and find that the tariff equivalent of the sunk exporting costs is around 30 percentage points. Antras & Caballero (2010) study long-run effects of trade liberalization with a dy-

[10] The elasticity of the price with respect to the terms of trade is the rate of exchange rate pass-through. Incomplete exchange rate pass-through arise when the movement of international relative prices tend to have a smaller impact on the price of imports.

namic general equilibrium model that incorporates financial constraints and the saving rate. Bernard et al. (2003) build a dynamic model with Bertrand competition in which heterogeneous firms are competing in prices and markups respond endogenously to these prices. In simulation results, they find that a 5 percent reduction of trade barriers lead to 4 percent increase in aggregate productivity and 4.7 percent increase in gross job creation. In opposition to the approaches taken in the papers above, this current study focuses on the aggregate effects of trade restrictions as a short-run feedback to economic slump of trading partner.

As a quantitative study, I start by analyzing the impulse response of the aggregate variables to temporary, negative productivity shock in the home economy. When the home economy is in an economic downturn, consumption and GDP go down. Its demand for varieties reduces with negative productivity shock and fewer firms enter the home market than before. Reduced entry in the home market generate less competition among firms, markups for all producers increase, and the cut-off productivity of home exporting firms increases since exporting becomes more difficult than before. Foreign producers exporting to the home economy become relatively competitive, so lower their markups and increase in exporting profits. This allows even less productive foreign firms can enter the home market. Therefore, the cut-off productivity of foreign exporting firms decrease during a recession of its trading partner and the terms of trade for home economy depreciate. Next, I analyze the consequences of the trade restrictions imposed by the foreign economy to protect its domestic industries as a response to economic downturn of its trading partner. The results show that both analyzed economies end up in a position worse than the one they would have found themselves in otherwise. The terms of trade for the home economy further depreciates, while consumption and income for both economies also continues to decrease. In the foreign economy, firms respond to this trade policy change in a number of ways. The profits of firms selling domestically increase and their markups go down, but the profits of exporting firms decrease and their markups increase with trade restrictions. However, the loss of profits of the exporting firms and the consumers in the foreign economy far outweigh the gains of the domestic pro ts, and put itself into a less competitive position than it was during the economic slowdown of its trading partner.

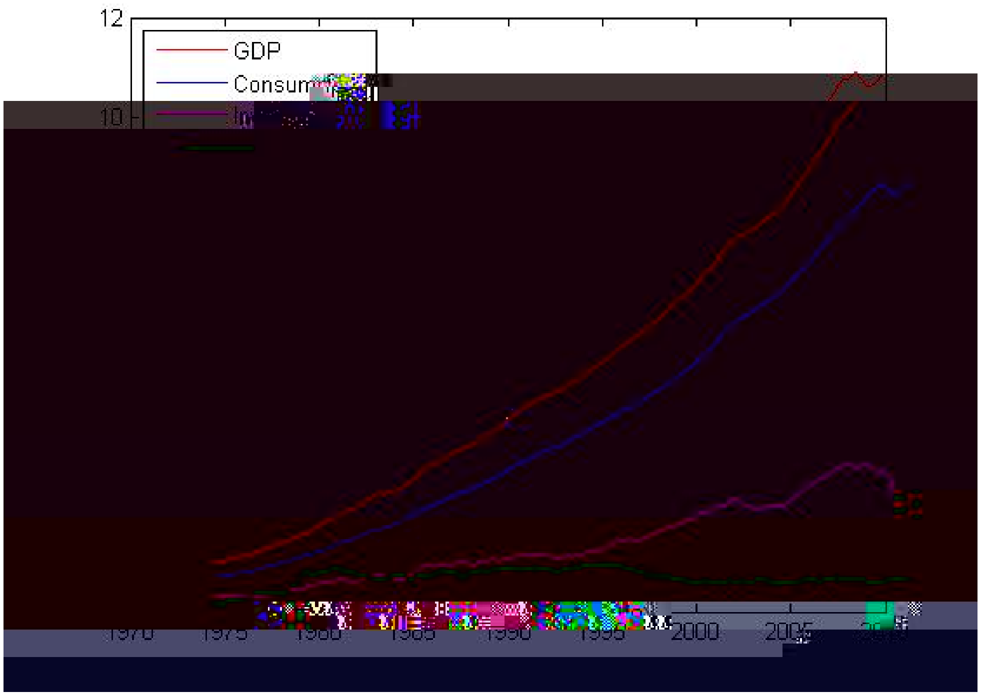
Second, international business cycle statistics of the simulated model are analyzed with a 1 percent home aggregate productivity shock, and with calibration along the lines of trade micro literature. Aggregate volatilities are well observed as a simillar pattern as the data. For the correlation between a variable and GDP, domestic comovement matches well, except for counter-cyclical net export. The average pro ts is positively correlated and markup is negatively correlated with GDP. These are the feature of the data that is in line with empirical ndings of Bilbiie et al. (2008). Regarding international correlations, the results shares the same failure of the conventional IRBC model. The model produces higher cross-country consumption correlations than output correlations. Also, the international correlations of labor and entry are strongly negatively correlated. However, due to the setting of the incomplete asset market, risk sharing between countries dampens demands of the goods, so international correlations of output is not strongly negatively correlated compared to conventional IRBC models and the relative consumption increase. It helps replicating the correlation between international relative prices and the consumption ratio across countries.

The paper is organized as follows. Section 2 provides stylized facts of international business cycle data. Section 3 describes the benchmark model that incorporates heterogeneous rms with selection to export and variable adjustment of markup in an incomplete asset market setting. Section 4 is the quantitative analysis, providing calibration, the transition dynamics of the economic slump and import restrictions, and international business cycle statistics of the model compared with data. Section 5 performs a sensitivity analysis, varying several key mechanisms of the model. Section 6 concludes.

## 2 Stylized Facts of International Business Cycle Data

This section provide stylized facts on the international business cycle data. I start by plotting the time series for GDP, consumption, investment, and labor for the U.S. over the sample post-Bretton Woods period, 1973Q1-2009Q4[1] . The time series plots are shown in Figure 3. The time series displays large uctuations about its trend at shorter frequencies, and consumption, investment, and labor time series comove with the GDP series. To make a comparison of the model dynamics with the business cycle properties of the data, cyclical components of the data needs to be extracted. As in the analysis by King & Rebelo (1999) and Backus et al. (1992), the Hodrick & Prescott (1997) lter[2] with a smoothing parameter equal to 1600 is applied to the natural log of each series.

Figure 3: Times Series using U.S. data




consumption as an other correlation. Cross-country output correlations (0.55) is larger than cross-country consumption correlations (0.42). Conventional IRBC models produce higher consumption correlations than output correlations. Investment and labor tend to be positively correlated across countries (0.39 and 0.28, respectively) in the data. The standard models fail to account for this feature and have counter-factually negative international correlations of investment and labor. Last, the terms of trade and the ratio of consumption

Table 1: U.S. Business Cycle Statistics (1973Q1-2009Q4)

| | Volatility | Domestic Comovement |
|---|---|---|
| | % S.D. relative to GDP | Correlations with GDP |
| GDP | 1 | 1 |
| Consumption | 0.72 | 0.86 |
| Investment | 3.87 | 0.89 |
| Employment | 0.58 | 0.79 |
| TOT | 1.44 | -0.25 |

countries include: Austria, Finland, France, Germany, Italy, Sweden, Switzerland and the U.K.

are negatively linked in the data (-0.35)[14] , but standard setups wrongly predict that they should be positively linked.

Table 2: International Correlations and Other Correlation (1973Q1-2008Q3)

| | |
|---|---|
| GDP, GDP | 0.55 |
| C, C | 0.42 |
| X, X | 0.39 |
| L, L | 0.28 |
| TOT, Relative Consumption | -0.35 (CDL) |

To evaluate the success and failure of the model, the data in this section and the simulated model is compared in the section 4.

# 3 A Model with Firm Selection and Variable Markup

In this section, I present a two-country, dynamic, stochastic, general equilibrium (DSGE) model that contains firm selection and variable adjustment of markup. The basic framework is built upon the models of Bergin & Glick (2007) and Ghironi & Melitz (2005) in which producers have heterogeneous firm-specific productivity and endogenous export participation with a sunk entry cost, and an ice-berg trade cost. The variable markups are introduced by non-homothetic preference of Melitz & Ottaviano (2008) that gives linear demand system for differentiated varieties. The world economy consists of two countries of equal size, home and foreign. The foreign variables are donoted by . The model economy is composed of infinitely lived representative households, perfectly competitive final goods producers, and monopolistically competitive intermediate goods producers. I assume that international financial markets are incomplete, allowing only for trade in uncontingent home and foreign bonds. I restrict attention to the behaviors of domestic agents unless otherwise necessary.

[14] This data is taken from Corsetti, Dedola & Leduc (2008)

## 3.1 The Household's Behavior

In each period, the representative household of each country supplies $L$ ($L^*$) units of labor inelastically at the wage rate $W_t$ ($W_t^*$). The expected intertemporal utility function is characterized by: $E_0 \sum_{t=0}^{1} {}^{t} \frac{C_t^{1-}}{1-}$ where $C_t$ denotes consumption. Here, the parameter $\in (0;1)$ is the intertemporal discount factor and $> 0$ is the inverse of the intertemporal elasticity of substitution. A unit mass of households in the home country face the sequence of budget constraints,

$$P_tC_t + P_tB_{H;t+1} + P_t^* B_{F;t+1} + \frac{n}{2} P_tB^2_{H;t+1} + P_t^* B^2_{F;t+1} + v_t(N_{A;t} + N_{E;t})q_{t+1}$$
$$= (1 + i_t)P_tB_{H;t} + (1 + i_t^*)P_t^* B_{F;t} + N_{A;t}(d_t + v_t)q_t + W_tL + {}_t \quad (1)$$

where $P_t$ denotes welfare-based price. $B_{H;t}$ and $B_{F;t}$ are home and foreign bond holdings in which pay an interest rate $i_t$ and $i_t^*$ each. Here, $W_tL$ is the income from labor and $W_t$ is equilibrium. Similarly, foreign households face the following sequence of budget constraints:

$$P_t C_t + P_t B_{F;t+1} + P_t B_{H;t+1} + \frac{n}{2} \left( P_t B^2_{F;t+1} + P_t B^2_{H;t+1} \right) + v_t (N_{A;t} + N_{E;t}) q_{t+1}$$
$$= (1 + i_t) P_t B_{F;t} + (1 + i_t) P_t B_{H;t} + N_{A;t}($$

and Ottaviano et al. (2002):

$$F_t = \alpha \int_{i \in \Omega} f_t(i)di - \frac{\gamma}{2}\int_{i \in \Omega}[f_t(i)]^2 di - \frac{\eta}{2}\left(\int_{i \in \Omega} f_t(i)di\right)^2 : \quad (8)$$

Here, $F_t$ is the production of final goods and $f_t(i)$ is the demand for varieties. $i \in \Omega$ denotes a continuum of differentiated varieties. I assume there is no homogeneous good in the preference[15]. Here, $\alpha$ measures the strength of the preference for differentiated products and $\eta$ governs the substitutability of varieties. $\gamma$ is a product differentiation index between intermediate goods in which consumers care more about the distribution of production across varieties as $\gamma$ increases[16]. The solution to this problem gives the linear demand function for each variety:

$$f_t(i) = \frac{\alpha}{\gamma} - \frac{1}{\gamma}\frac{p_t(i)}{P_t} - \frac{\eta}{\gamma}\int_{i \in \Omega} f_t(i)di: \quad (9)$$

In the home economy, total number of producers are $N_t$. Therefore, all the varieties produced in home economy is achieved integrating (9) over $N_t$:

$$\begin{aligned}
\int_{i \in \Omega} f_t(i)di &= N_t\frac{\alpha}{\gamma} - \frac{1}{\gamma}\frac{1}{P_t}\int_{i \in \Omega} p_t(i)di - \frac{\eta}{\gamma}N_t\int_{i \in \Omega} f_t(i)di \\
&= \frac{\gamma}{\gamma + \eta N_t}\left[\frac{\alpha N_t}{\gamma} - \frac{1}{\gamma}\frac{1}{P_t}\int_{i \in \Omega} p_t(i)di\right] \\
&= \frac{\alpha N_t}{\gamma + \eta N_t} - \frac{N_t \bar{p}_t}{P_t(\gamma + \eta N_t)}
\end{aligned}$$

where $\bar{p}_t = \frac{1}{N_t}\int_{i \in \Omega} p_t(i)di$. Now, plugging this to (9) gives the expression for the variety demand without integral:

$$f_t(i) = \frac{\alpha}{\gamma} - \frac{1}{\gamma}\frac{p_t(i)}{P_t} - \frac{\eta}{\gamma}\frac{\alpha N_t}{\gamma + \eta N_t} + \frac{\eta}{\gamma}\frac{N_t \bar{p}_t}{P_t(\gamma + \eta N_t)}: \quad (10)$$

The price bound, $p_{bound;t}$, is attained at which linear demand for each variety, $f_t(i)$ is driven to 0. If price is lower than $p_{bound;t}$, a firm would have zero demand. This price bound is the

[15] In Melitz & Ottaviano (2008), preference includes a homegenous good $f_0$ chosen as numeraire.
[16] When $\gamma$ is zero, differentiated varieties are perfect substitutes.

Producers maximize their profits separately and decide how much to produce on each market. Producers selling domestically maximize $d_{D;t}(a) = p_{D;t}(a) f_{D;t}(a) \quad \frac{W_t}{aZ_t} f_{D;t}(a)$ subject to

$$f_{D;t}(a) = - \quad \frac{p_{D;t}(a)}{P_t} \quad - \quad \frac{N_t}{+ \; N_t} \quad + - \frac{N_t p_t}{P_t( \; + \; N_t)} \tag{13}$$

while exporting producers maximize $d_{X;t}(a) = p_{X;t}(a) f_{X;t}(a) \quad \frac{W_t}{aZ_t} \;_t f_{X;t}(a)$ subject to

$$f_{X;t}(a) = - \quad \frac{p_{X;t}(a)}{P_t} \quad - \quad \frac{N_t}{\phantom{N_t}}$$

Here, $p_{bound;t}$ is defined as the price bound for the producers who are having domestic sales. If its price is lower than $p_{bound;t}$, a firm would have zero demand. Therefore, it is the threshold cost for the firms who are having domestic sales, and is equal to $p_{D;t}(a_{D;t})$ and $\frac{W_t}{a_{D;t} Z_t}$. Similarly, the price bound of producers who have export sales, $p_{bound;t}$ is defined when $f_{X;t}(a_{X;t})$ is zero. Therefore, it is the threshold cost for the firms who are having export sales, and is equal to $p_{X;t}(a_{X;t})$ and $\frac{W_t}{a_{X;t} Z_t}$.

Since demand functions are written in the function of the price function, I plug optimal prices and the threshold cost for the producers back into demand function and yield:

$$f_{D;t}(a) = \frac{1}{P_t}\left[\frac{\frac{W_t}{a_{D;t} Z_t} - \frac{W_t}{aZ_t}}{2}\right] \tag{17}$$

and

$$f_{X;t}(a) = \frac{1}{P_t}\left[\frac{\tau_t\frac{W_t}{a_{X;t} Z_t} - \frac{W_t}{aZ_t}\tau_t}{2}\right]: \tag{18}$$

As in the optimal prices, demand functions of the producers are bounded from above and determined by the cut-off productivity strategy.

### 3.3.2 Markups and Profits

The monopolistically competitive producers have excess capacity in which they operate on the downward sloping portion of their average total cost curve. Therefore, they produce less than the cost-minimizing output and have markup over marginal cost. The exogeneous markup is a common form in the IRBC models, because the good is aggregated using the constant elasticity of substitution (CES) technology. In this paper, the endogenous adjustment of markups of producers is generated from the variable elasticity of substitution (VES) technology of the final goods that aggregates a continuum of horizontally differentiated intermediate goods. Plugging the optimal pricing rules, $p_{D;t}(a)$ and $p_{X;t}(a)$ into markup formula, the expressions for markup are as follow.

$$\mathrm{mu}_{D;t}(a) = \mathrm{p}_{D;t}(a) \quad \frac{W_t}{aZ_t} = \frac{\frac{W_t}{a_{D;t}\ Z_t} \qquad \frac{W_t}{aZ_t}}{2} \tag{19}$$

$$\mathrm{mu}_{X;t}(a) = \mathrm{p}_{X;t}(a) \quad \frac{W_t}{aZ_t} = \frac{t\,\frac{W_t}{a_{X;t}\ Z_t} \qquad \frac{W_t}{aZ_t}}{2} \tag{20}$$

Similarly, the pro ts of domestic sales $d_{D;t}(a)$ and exporting sales $d_{X;t}(a)$ are found by plugging in the optimal pricing rules $p_{D;t}(a)$ and $p_{X;t}(a)$ and the demand functions $f_{D;t}(a)$ and $f_{X;t}($a

from domestic sales and export sales is found using the de nition of average productivities: $đ_{D;t} = \frac{1}{2 P_t(1 \; 2)} \frac{a_{min}}{a_{D;t}} \frac{W_t}{Z_t a_{D;t}}^2$ and $đ_{X;t} = \frac{1}{2 P_t(1 \; 2)} \frac{a_{min}}{a_{X;t}} \frac{W_t \; t}{Z_t a_{X;t}}^2$. Aggregating technology of the nal goods, $F_t$ yields:

$$
\begin{aligned}
F_t = & \int_{i2} f_t(i)di \quad \frac{}{2} \int_{i2} [f_t(i)]^2 di \quad \frac{}{2} \int_{i2} f_t(i)di^{2} \\
= & \frac{N_t}{2(\;+1)P_t} \frac{W_t}{Z_t a_{D;t}} \quad \frac{N_t}{4(\;+1)(\;+2)P_t^2} \frac{W_t}{Z_t a_{D;t}}^2 \\
& \frac{}{2} \frac{N_t}{2(\;+1)P_t}^2 \frac{W_t}{Z_t a_{D;t}}^2 : \qquad (26)
\end{aligned}
$$

## 3.6 Market Clearing Conditions and Equilibrium

The quilibrium for the benchmark model requires several market-clearing conditions. Firstly, the nal goods produced, $F_t$ in the economy are all consumed by households. Therefore, $F_t = C_t$. The model is closed by the bond market clearing conditions $B_{H;t1} + B_{H;t1} = 0$ and $B_{F;t1} + B_{F;t1} = 0$ as well as by the value of shares in a mutual fund market clearing condition $q_{t1} = q_{t1} = 1$. Subtracting foreign household's budget constraints (2) from the budget constraints of household in the home economy (1) and then applying the bond and mutual fund market clearing conditions gives the net foreign assets condition as follows.

$$
\begin{aligned}
P_t B_{H;t1} + P_t B_{F;t1} = & P_t(1+i_t)B_{H;t} + P_t(1+i_t)B_{F;t} + \frac{1}{2}(W_t L \quad W_t L) \\
& \frac{1}{2}(P_t C_t \quad P_t C_t) + \frac{1}{2}\; N_{A;t} đ_t \quad N_{A;t} đ_t \quad \frac{1}{2}\; N_{E;t} \forall_t \quad N_{E;t} \forall_t \qquad (27)
\end{aligned}
$$

Finally, the labor market clearing condition requires that labor employed in domestic production and exporting production, and labor employed to cover the entry costs equal the xed labor supply $L$:

$$
\begin{aligned}
L &= \frac{}{W_t} đ_{D;t} N_{D;t} \frac{1}{1 \quad (a_{D;t})} + \frac{}{W_t} đ_{X;t} N_{X;t} \frac{1}{1 \quad (a_{X;t})} + \frac{N_{E;t} f_{E;t}}{Z_t} \\
&= \frac{}{2 \ ( \quad + 1)( \quad + 2) P_t W_t} \quad \frac{W_t}{a_{D;t} Z_t}^{\ 2} N_{D;t} \\
&\quad + \frac{}{2 \ ( \quad + 1)( \quad + 2) P_t W_t} \quad \frac{W_t \ _t}{a_{X;t} Z_t}^{\ 2} N_{X;t} + \frac{N_{E;t} f_{E;t}}{Z_t}
\end{aligned}
\tag{28}
$$

The benchmark model economy and its associated steady state system has 45 independent equations, so 45 variables must be solved for: 23 home variables ( $_t$; $C_t$; $W_t$; $i_t$; $P_t$; $đ_t$; $v_t$; $N_{A;t}$; $N_{D;t}$; $N_{X;t}$; $N_{E;t}$; $p_t$; $p_{D;t}$; $p_{X;t}$; $mu_{D;t}$; $mu_{X;t}$; $a_{D;t}$; $a_{X;t}$; $N_t$; $đ_{D;t}$; $đ_{X;t}$; $B_{H;t}$; $B_{F;t}$) and 22 foreign variables ( $_t$; $C_t$; $i_t$; $đ_t$; $v_t$; $N_{A;t}$; $N_{D;t}$; $N_{X;t}$; $N_{E;t}$; $p_{D;t}$; $p_{X;t}$; $mu_{D;t}$; $mu_{X;t}$; $a_{D;t}$; $a_{X;t}$; $N_t$; $đ_{D;t}$; $đ_{X;t}$; $W_t$ ¡ pp of shocks, DYNARE with MATLAB program[18] are used to solve and simulate a system of linear di erence equations.

## 4.1 Benchmark Calibration

The benchmark values are chosen for the set of relevant parameters to match the features of the US economy. A standard choice in the literatures, the intertemporal discount factor of households is set equal to 0.99. The inverse of the intertemporal elasticity of substitution is set equal to 2 as in Ghironi & Melitz (2005) and the quadratic adjustment cost of bond holdings is set equal to n = $^2$ 0:01 as in Boileau & Normandin (2008). Following closely with Sakane (2011) and Rodriguez-Lopez (2011), I set the technology of the nal goods parameters as =9.5, =0.5, and =1.1. Relying on Chaney (2008), the scaling parameter of the Pareto distribution condition holds in order to assure the standard deviation of the idiosyncratic shock is nite and positive. As documented by Bernard et al. (2003), this paramter also matches the standard deviation of the log of domestic US plant sales at 1.67 in a steady state. I set the probability of a death shock equal to 0.025, which implies that average annual death rate for US rms is 10%. As in Alessandria & Choi (2007) and Obstfeld & Rogo (2000), I set the steady-state value of ice-berg transport cost equal to 1.4, and the steady-state value of the entry cost is 1 as in Ghironi & Melitz (2005). Labor endowment is normalized to 1 for both economies. The minimum value of the productivity, $a_{min}$ is also set equal to 1, without loss of generality. The steady state cut-o productivity for produers who sell in domestic market, $a_D$ is found solving the symmetric steady-state equilibrium. Table 3 lists all calibrated parameters.

[18] I simulate the model using Dynare version 4.2.4. See Juillard (2001).

Table 3: Benchmark Parameter Values

| Description | value |
|---|---|
| Strength of product di erentiation coe cient | $=9.5$ |
| Product di erentiation index | $=0.5$ |
| Variety substitutability | $=1.1$ |
| Inverse of the intertemporal elasticity of substitution | $=2$ |
| Intertemporal discount factor | $=0.99$ |
| Probability of death shock | $_d=0.025$ |
| Ice-berg transport cost | $=1.4$ |
| Sunk entry costs parameter | $f_E=1$ |
| Quadratic adjustment cost of bond holdings | $n = \ ^2 \ 0{:}01$ |
| Cut-o productivity for domestic rms | $a_D=1.793$ |
| Lower bound of productivity | $a_{min}=1$ |
| Characterizing parameter of (a) | $=3.4$ |
| Labor endowment | $L = L = 1$ |

## 4.2 Shocks Strategy

### 4.2.1 Productivity Shocks

I solve for the dynamics in response to deterministic and stochastic shocks by log-linearizing the model around the steady state. In order to analyze the consequences of the economic slump in the home economy, a deterministic and negative shock to aggregate prodcuctivity in the order of 1 percent deviations from the steady-state value is considered. This deterministic shock is only allowed to be temporary (duration of the shock is one year), and the model eventually comes back to the steady state. The shock process is to study the impact of a change in regime, as home economy falls into recession.

In order to analyze the business cycle statistics, stochastic shocks to aggregate productivities are introduced. The positive shocks hit unexpectedly. For this, I use a bivariate autoregressive process for percent deviations of home and foreign aggregate productivities from their steady state. The symmetric and exogenous process can be expressed as follows (in the log-linearized form):

$$\begin{bmatrix} z_t \\ \tilde{z}_t \end{bmatrix} = \begin{bmatrix} \rho & \rho_{HF} \\ \rho_{FH} & \tilde{\rho} \end{bmatrix} \begin{bmatrix} z_{t-1} \\ \tilde{z}_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ \tilde{\varepsilon}_t \end{bmatrix} \text{ with } \begin{bmatrix} \varepsilon_t \\ \tilde{\varepsilon}_t \end{bmatrix} \sim N\left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \begin{bmatrix} \sigma_{\varepsilon}^2 & \sigma_{\varepsilon\tilde{\varepsilon}} \\ \sigma_{\varepsilon\tilde{\varepsilon}} & \sigma_{\tilde{\varepsilon}}^2 \end{bmatrix} \right)$$

As in Backus et al. (1994), the persistence of the aggregate productivity shock ($\rho$, $\tilde{\rho}$) is set to 0.906. The spill over parameter $\rho_{HF}$, $\rho_{FH}$ is set to 0.088. The standard deviation of the productivity innovations is 0.00852 and the correlation between productivity innovations is 0.258.

Under permanent productivity shocks, the model reaches a new steady state and shocks are entirely expected. To study the effects of permanent productivity shocks hitting the economy today, the initial and ending values are set so as to calculate the transition path of each key variables. Since the results of the deterministic and permanent productivity shocks are similar to the one from stochastic productivity shocks, the resulting impulse response functions are only illustrated in the Appendix.

Larch & Lechthaler (2011), a simple trade restriction setting rule is generated as follow:

$$1 + \ {}_{t} = (1 + \ )\frac{Z}{Z_t} \tag{29}$$

and

$$1 + f_{E;t} = (1 + f_E)\frac{Z}{Z_t} \tag{30}$$

This trade shock process shows that as trade costs or entry costs decrease by 1 percent, aggregate productivity increases by 1 percent, and vice versa.

## 4.3 Macroeconomic Dynamics

In this subsection, the dynamics of a recession and trade restrictions are thoroughly analyzed. First, I begin by analyzing the follow-up to a recession in home country. After that, the subsequent introduction of trade restrictions in foreign economy is analyzed. The trade restrictions is imposed by foreign economy to protect its domestic industries that got hurt from the spillover of the home country's economic downturn through the interconnection of trade.

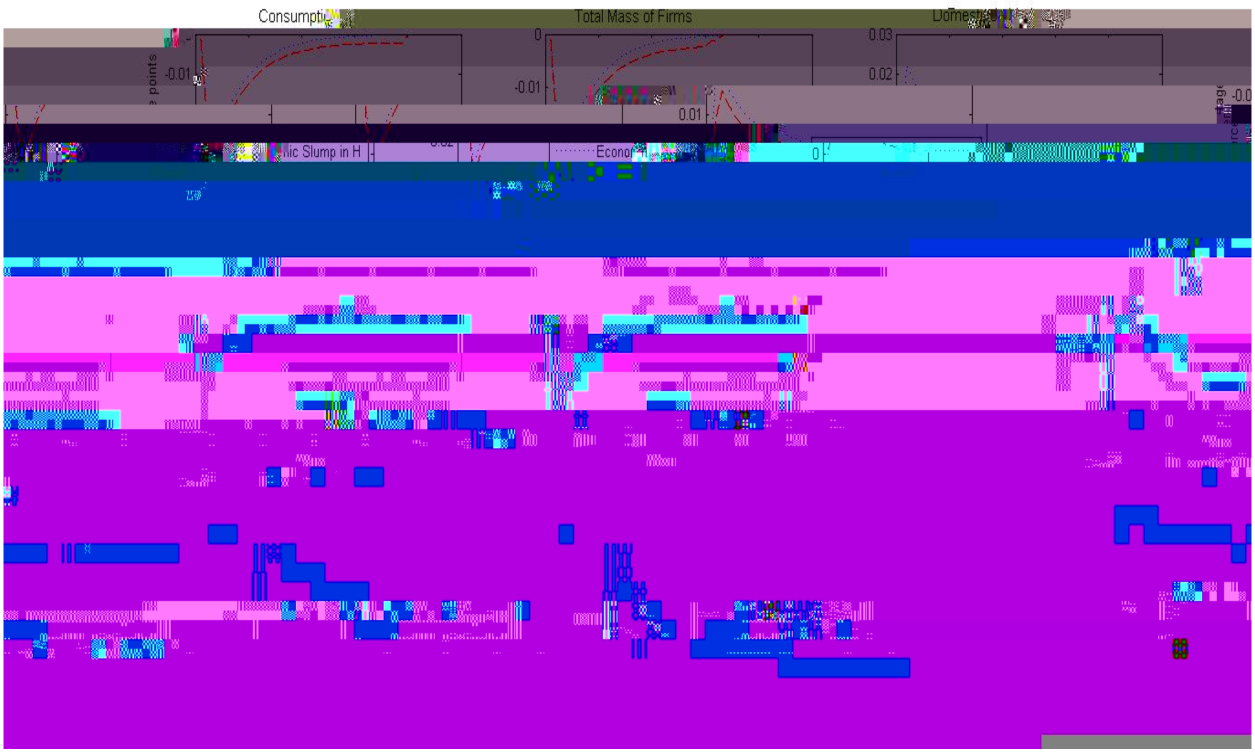
### 4.3.1 Economic Slump

The  rst case is that of an economic downturn in the home economy. The economy starts from the stationary steady-state and a 1 percent exogenous, asymmetric, temporary, and negative productivity shock hits the home economy. The dynamic responses of main variables to this shock are illustrated in Figure 4 (home) and Figure 5 (foreign). The duration of the shock is one year and the horizontal axis on the impulse responses is the number of years after shock. The negative shock leads to a depression in the home economy. Not surprisingly, productivity falls (Z #). The economic slump also matters to the number of producers. Now, the home market is relatively less competitive than before and the number of newly created rms has decreased ($N_E$ #). This leads the total number of producers to fall as well (N #) in the home economy. As previously described, rms' markups are the avenue of `toughness' of competition and more competitive rms lower their markups in the micro rm-level dynamics. The result macroeconomic dynamics show that producers' variable markups have e ect on aggregate uctuations. Since the home market is less competitive than before, markups for home producers in domestic and exporting markets increase ($\bar{m}u_D$ ", $\bar{m}u_X$ ")"

smaller than in the home economy. Because of the economic downturn in the home economy, fewer home producers export to foreign country ($N_X \downarrow$) and this leads the total number of producers to fall ($N \downarrow$) as well in the foreign economy ($N = N_D + N_X$). Interestingly, due to the fact that the home market is less competitive, foreign producers exporting to home economy become relatively competitive and decrease their markups ($\tilde{\mu}_X \downarrow$). Consequently, the average profit of foreign exporting firms increases during the shock ($\tilde{d}_X \uparrow$). The increase in exporting profit in the foreign country makes them being relatively more productive than home exporting firms as their cut-off productivity decreases ($\tilde{a}_X \downarrow$). It means that relatively less productive foreign firms are able to export to home economy. In contrast, demands for in the trade balance of a country, the terms of trade is de ned as the ratio of the price of imports to the price of exports ($TOT = \frac{P_{IMP}}{P_{EXP}}$). The real exchange rate is de ned as the ratio of the price index of the nal goods ($RER = \&\frac{P}{P}$). During a recession of the home economy, the price of home exports become cheaper and the terms of trade and real

The blue dotted line represents the case of the economic slump in the home economy and the red dashed line represents the case of the trade restrictions imposed by the foreign economy. The trade cost or entry cost of home exporting firms to foreign economy only increased due to this change in trade policy. Since the home economy does not raise its trade restriction, the trade cost or entry cost for foreign exporting firms to the home economy does not change. Also, I assume that this imposed trade restrictions does not have any direct effect on foreign government revenue. Therefore, increase in trade cost can be understood as any types of non-tariff barriers such as a voluntary export restraint (VER), `Buy national' policy, quota shares, or export subsidies.

Figure 6: Trade Restrictions by Foreign: Home Economy




Surprisingly, the increase in trade restrictions in the foreign economy is followed by a further decrease in consumption ($C; C^{\#}$) and income ($GDP; GDP^{\#}$) in both countries. As shown in the dynamic responses, consumption in the foreign economy drop sharply while consumption in the home economy decline slightly. This change in policy harm home and foreign consumers because of the increase in prices in the foreign country. Due to the trade limitation on home exports, the number of home exporting rms and their average pro ts further go down ($N_X$ #, đ$_X$ #). This clearly shows through the further increase in the cut-o productivity of home exporting rms ($a_X$ ") since exporting become di cult for them due to the trade barrier. In the foreign country, the trade limiting-measures lead to diverse results for domestically selling rms and exporting rms. Since domestic industries are shielded from cheap imports, they become competitive and markups actually go down ($mu_D$ #). Consequently, their pro ts increase pror2X

for domestically selling rms and and and and and and).

Tf 510.071 Td [())-27114rms)-422(and)).

raise their markup ($\overline{mu}_X$ ″) more than before. This lead to further decrease in exporting pro ts ($\bar{\pi}_X$ #). This pushes their price level lower than before (P #) and its export price further go down ($P_{EXP}$ #). This makes the real exchange rate and the terms of trade in home economy depreciate more ($RER; TOT$ ″) with implementation of trade restrictions of the foreign economy. The markup for producers selling in domestic market increase ($\overline{mu}_D$ ″) and their average pro ts still decrease ($\bar{\pi}_X$ #), but less magnitude than economic slump in the economy.

In the foreign country, lower GDP and consumption, further appreciation of the international relative prices, a sharp decrease in average export pro ts, and increasing markup for exporting industries counteracts the reduced markup and increased average pro ts of domestically selling rms. These e ects clealy show that trade restrictions not only hurt the trading partner, but also the country imposing them damaging its market competitiveness even though its domestic industries are protected from lower prices of imports. In summary, foreign country impose trade restrictions to protect its domestic industries that got hurt mostly from the recession of its trading partner. The policy bene t domestically producing and selling producers, but harm consumers and exporting producers in the economy. The losses to the trade restrictions far outweigh the gains, and analyzed economy end up worse o than they would be otherwise during the economic downturn of the home economy.

## 4.4 International Real Business Cycle Moments

To further evaluate the properties of the simulated model, business cycle statistics of the simulated model are computed with a stochastic shock to the aggregate productivity in the home economy. I augment the benchmark model (as in section 3) with elastic labor. Here, unconditional second moments are presented using the benchmark model and comparing this to what is observed in the economic data for the US and European countries (See section 2). I use the model to confront the observations on business cycle statistics. The Hodrick and

Table 4: Business Cycle Statistics: Baseline Parameters

| | Data | CES | Inelastic Labor | Benchmark | |
|---|---|---|---|---|---|
| | | IM | IM | IM | FA |
| Volatility | | | | | |
| % S.D. relative to GDP | | | | | |
| GDP | 1 | 1 | 1 | 1 | 1 |
| Consumption (C) | 0.72 | 0.52 | 0.41 | 0.32 | 1 |
| Employment (L) | 0.58 | 0.58 | – | 0.18 | 0.25 |
| Investment (X) | 3.87 | 2.99 | – | – | – |
| Net Export (NX/Y) | 0.45 (BKK) | – | 0.43 | 0.58 | – |
| Terms of Trade (TOT) | 1.44 | 0.32 | 0.38 | 0.38 | 0.08 |
| Entry ($N_E$) | – | 1.69 | 4.39 | 4.40 | 3.72 |
| Domestic Comovement | | | | | |
| Correlations with GDP | | | | | |
| Consumption (C) | 0.86 | 0.70 | 0.42 | 0.22 | 1 |
| Employment (L) | 0.79 | 0.61 | – | 0.68 | 0.68 |
| Net Export (NX/Y) | -0.47 (BKK) | – | 0.73 | 0.64 | – |
| Terms of Trade (TOT) | -0.25 | -0.53 | -0.46 | -0.48 | 0.58 |
| Entry ($N_E$) | – | 0.51 | 0.52 | 0.52 | 0.49 |
| Mark-up ($\tilde{MU}$) | – | – | -0.90 | -0.91 | -0.89 |
| Average Profits ($\tilde{d}$) | – | – | 0.53 | 0.53 | 0.47 |
| International Correlations | | | | | |
| GDP, GDP | 0.55 | -0.87 | -0.23 | -0.21 | 0.10 |
| C, C | 0.42 | 0.21 | 0.06 | 0.06 | 0.10 |
| X, X | 0.39 | -0.89 | – | – | – |
| L, L | 0.28 | -0.23 | – | -0.91 | 0.65 |
| $N_E$, $N_E$ | – | -0.84 | -0.92 | -0.92 | -0.81 |
| Other Correlation | | | | | |
| Consumption ratio, TOT | -0.35 (CDL) | -0.93 | -0.37 | -0.39 | 0.18 |

cyclicity with GDP, except net exports. Consumption (0.22), employment (0.68), entry (0.52), and average profits (0.53) are positively correlated and terms of trade (-0.48) and the standard IRBC models and adding entry and exit dynamics along with rm selection to the benchmark model does not help. The model also fails to predict the higher cross-country GDP correlations than consumption correlations (what Backus et al. (1992) call `quantity

## 5.1 Exogenous Markup (CES preference)

Based on the model of Bergin & Glick (2007) and Ghironi & Melitz (2005), Moon (2012) studies international relative prices and endogenous tradability, incorporating endogeneous labor and capital along the IRBC setting. The technology of the nal goods is that combines home and foreign produced intermediate goods as in Armington (1969):

$$F_t = \left[ \left( \int_{a \in {}_{D;t}} f_{D;t}(a)^{\frac{\ \ 1}{\ }} da \right)^{\frac{\ }{\ 1}\frac{\ 1}{\ }} + \left( \int_{a \in {}_{X;t}} f_{X;t}(a)^{\frac{\ \ 1}{\ }} da \right)^{\frac{\ }{\ 1}\frac{\ 1}{\ }} \right]^{\frac{\ }{\ 1}} \qquad (31)$$

where is the elasticity of substitution between domestic and foreign varieties of intermediate goods, and is the elasticity of substitution among domestic varieties. Dixit & Stiglitz (1977) refer to as a `love of variety' parameter in which, when more varieties are available, more goods are produced, and more consumers are satis ed.

## 5.2 Financial Autarky

Endogenizing labor, the utility function of the representative households is characterized by:

$$E_0 \sum_{t=0}^{\infty} {}^{t} \frac{\{C_t (1 \quad L_t)^{1} \quad \}^{1}}{1}$$

where $C_t$ denotes consumption, and $L_t$ represents hours worked. Here, the parameter is the intertemporal discount factor, is the consumption weights in utility, and is the coe cient of relative risk aversion. In the case of the nancial autarky, the buget constraint is as follows:

$P_t C_t + P_t B$

The Euler equation for bond holdings is

$$[C_t (1 \quad L_t)^{1} \quad ]^{1} \quad \frac{1}{C_t} = \quad (1 + i_{t1})E_t \quad [C_{t1} \; (1 \quad L_{t1})^{1} \quad ]^{1} \quad \frac{1}{C_{t1}} \quad : \tag{33}$$

The Euler equation for the shares in a mutual fund is

$$\forall_t = \quad (1 \qquad )E_t \quad \frac{P_t C_t}{P_{t1} \; C_{t1}} \quad \frac{[C_{t1} \; (1 \quad L_{t1})^{1} \quad ]^{1}}{[C_t (1 \quad L_t)^{1} \quad ]^{1}} (\text{đ}_{t1} \quad + \forall_{t1}) \quad : \tag{34}$$

The nancial autarky model is closed by the bond market clearing condition $B_{t1} \quad = B_{t1}$

# 6 Concluding Remarks

This paper explored the aggregate effects of an economic slump and trade restrictions as a short-run response along international real business cycles. During the crisis of 2008 and 2009, world output, exports, and imports collapsed tremendously. As a response to global crisis, international trade-limiting measures have emerged in several countries. In order to capture the recession and the change in trade policy along the IRBC, I proposed a DSGE model with firm entry and exit dynamics, non-homothetic preferences of the final goods technology with product differentiation, and heterogeneity in firm productivity. The variable adjustment of markups was generated from the non-homothetic, non-constant elasticity of substitution production function of the final goods. By analyzing the dynamics of an economic slump in the home economy and then an increase in trade restrictions in the foreign economy as part of a policy to protect itself from the diffusion of recession, I showed that both economies are in a worse position than during the economic downturn. The follow-ups to the recession and trade restrictions were analyzed through the variable markups, firms' individual specific productivity cut-off, and the movement of international relative prices such as real exchange rate and terms of trade. The foreign country suffered from the economic downturn of its trading partner and imposed trade restrictions on import goods from the home economy. There were winners and losers from the implementation of the import restrictions, but the losses far outweighed the gains, and both analyzed economies ended up worse off than they would be.

The simulated model replicated several U.S. business cycle statistics and emphasized the fact that the endogenous entry of heterogeneous firms with various adjustment of markup may have important effects for the interpretation of the international transmission of business cycles. Possible future work will be to augment the model with banking sector, analyzing the effect of banking deregulation and to explore the ability of the model using quasilinear non-constant elasticity of substitution production function and heterogeneous producers.

# References


Alessandria, G. & Choi, H. (2007). Do sunk costs of exporting matter for net export dynamics? *The Quarterly Journal of Economics*, *122* (1), 289{336.

Alessandria, G. & Choi, H. (2011). Establishment heterogeneity, exporter dynamics, and the e ects of trade liberalization. Working Papers 11-19, Federal Reserve Bank of Philadelphia.

Amiti, M. & Weinstein, D. E. (2009). Exports and nancial shocks. NBER Working Papers 15556, National Bureau of Economic Research, Inc.

Antras, P. & Caballero, R. J. (2010). On the role of nancial frictions and the saving rate during trade liberalizations. *Journal of the European Economic Association*, *8*, 442{455.

Armington, P. S. (1969). A theory of demand for products distinguished by place of production. *IMF Sta Papers*, *16* (1), 159{178.

Arvanitis, A. V. & Mikkola, A. (1996). Asset-market structure and international trade dynamics. *American Economic Review*, *86* (2), 67{70.

Aw, B. Y., Chung, S., & Roberts, M. J. (2000). Productivity and turnover in the export market: Micro-level evidence from the republic of korea and taiwan (china). *World Bank Economic Review*, *14* (1), 65{90.

Backus, D. K., Kehoe, P. J., & Kydland, F. E. (1992). International real business cycles. *Journal of Political Economy*, *100* (4), 745{75.

Backus, D. K., Kehoe, P. J., & Kydland, F. E. (1994). Dynamics of the trade balance and the terms of trade: The j-curve? *American Economic Review*, *84* (1), 84{103.

Backus, D. K. & Smith, G. W. (1993). Consumption and real exchange rates in dynamic economies with non-traded goods. *Journal of International Economics*, *35* (3-4), 297{316.

Baxter, M. & Crucini, M. J. (1995). Business cycles and the asset structure of foreign trade. *International Economic Review*, *36*, 821{54.

Bergin, P. R. & Corsetti, G. (2008). The extensive margin and monetary policy. *Journal of Monetary Economics*, *55* (7), 1222{1237.

Bergin, P. R. & Glick, R. (2007). Tradability, productivity, and international economic integration. *Journal of International Economics*, *73* (1), 128{151.

Bernard, A. B. & Bradford Jensen, J. (1999). Exceptional exporter performance: cause, e ect, or both? *Journal of International Economics*, *47* (1), 1{25.

Bernard, A. B., Eaton, J., Jensen, J. B., & Kortum, S. (2003). Plants and productivity in international trade. *American Economic Review*, *93* (4), 1268{1290.

Bernard, A. B., Jensen, J. B., & Schott, P. K. (2006). Trade costs, rms and productivity. *Journal of Monetary Economics*, *53* (5), 917{937.

Bilbiie, F. O., Ghironi, F., & Melitz, M. J. (2008). Monetary policy and business cycles with endogenous entry and product variety. In *NBER Macroeconomics Annual 2007, Volume 22*, NBER Chapters (pp. 299{353). National Bureau of Economic Research, Inc.

Boileau, M. & Normandin, M. (2008). Closing international real business cycle models with restricted  nancial markets. *Journal of International Money and Finance*, *27* (5), 733{756.

Caballero, R. J. & Hammour, M. L. (1994). The cleansing e ect of recessions. *American Economic Review*, *84* (5), 1350{68.

Campbell, J. R. & Hopenhayn, H. A. (2005). Market size matters. *Journal of Industrial Economics*, *53* (1), 1{25.

Ottaviano, G., Tabuchi, T., & Thisse, J.-F. (2002). Agglomeration and trade revisited.

include Austria, Finrand, France, Germany, Italy, Sweden, Switzerland, and the U.K. Investment includes gross xed capital formation and changes in inventories. Labor input per capital is calculated as hours per worker multiplied by civilian employment and then devided by population age 16 and over. I follow the tradition of the international business cycle literature in de ning the terms of trade as the relative price of imports to exports.

# B The Set of Equations

## B.1 Benchmark Model - Incomplete Asset Market

I list summary of 45 equilibrium system of equations of the model.

Optimal conditions for Consumption

$$_tP_t = C_t \tag{B.1}$$

$$_t = C_t \tag{B.2}$$

Euler Equations (Bonds)

$_tP_t(1$(B.2)

$$N_{D;t} = (1 \quad (a_{D;t}))\, N_{A;t} \tag{B.15}$$

$$N_{X;t} = (1 \quad (a_{X;t}))\, N_{A;t} \tag{B.16}$$

$$N_{D;t} = \quad 1 \quad (a_{D;t}) \quad N_{A;t} \tag{B.17}$$

$$N_{X;t} = \quad 1 \quad (a_{X;t}) \quad N_{A;t} \tag{B.18}$$

Total Average Pro ts

$$\text{đ}_t = \text{đ}_{D;t} + \text{đ}_{X;t} \tag{B.19}$$

$$\text{đ}_t = \text{đ}_{D;t} + \text{đ}_{X;t} \tag{B.20}$$

Average Pro ts from Domestic Sales

$$\text{đ}_{D;t} = \frac{1}{2\,P_t(\quad+1)(\quad+2)} \quad \frac{a_{min}}{\bar{a}_{D;t}} \quad \frac{W_t}{Z_t\bar{a}_{D;t}}^{\,2} \tag{B.21}$$

$$\text{đ}_{D;t} = \frac{1}{2\;(\quad+1)(\quad+2)} \quad \frac{a_{min}}{\bar{a}_{D;t}}\,! \quad \frac{W_t}{Z_t\,\bar{a}_{D;t}}\,!^{\,2} \tag{B.22}$$

Average Pro ts from Foreign Sales

$$\text{đ}_{X;t} = \frac{1}{2\;(\quad+1)(\quad+2)} \quad \frac{a_{min}}{\bar{a}_{X;t}} \quad \frac{W_{t\ \ t}}{Z_t\bar{a}_{X;t}}^{\,2} : \tag{B.23}$$

$$\text{đ}_{X;t} = \frac{1}{2\,P_t(\quad+1)(\quad+2)} \quad \frac{a_{min}}{\bar{a}_{X;t}}\,! \quad \frac{W_{t\ \ t}}{Z_t\,\bar{a}_{X;t}}\,!^{\,2} \tag{B.24}$$

Price Bounds/Cost Threshold

$$\frac{W_t}{Z_t\bar{a}_{D;t}} = \frac{P\ \ _t + \ N\ _t\,\text{ᵽ}_t}{+\ \ N\ _t} \tag{B.25}$$

$$\frac{W_{t\ \ t}}{Z_t\bar{a}_{X;t}} = \frac{+\ \ N\ _t\,\text{ᵽ}_t}{+\ \ N\ _t} \tag{B.26}$$

$$\frac{W_t}{Z_t\,\bar{a}_{D;t}} = \frac{+\ \ N\ _t\,\text{ᵽ}_t}{+\ \ N\ _t} \tag{B.27}$$

$$\frac{W_{t\ \ t}}{Z_t\,\bar{a}_{X;t}} = \frac{P\ \ _t + \ N\ _t\,\text{ᵽ}_t}{+\ \ N\ _t} \tag{B.28}$$

Average Relative Prices

$$\text{ᵽ}_{D;t} = \frac{2\quad+1}{2\quad+2} \quad \frac{W_t}{Z_t\bar{a}_{D;t}} \tag{B.29}$$

$$\text{ᵽ}_{X;t} = \frac{2\quad+1}{2\quad+2} \quad \frac{W_{t\ \ t}}{Z_t\bar{a}_{X;t}} \tag{B.30}$$

$$\tilde{p}_{D;t} = \frac{2 \quad + 1}{2 \quad + 2} \left( \frac{W_t}{Z_t \, \tilde{a}_{D;t}} \right) \tag{B.31}$$

$$\tilde{p}_{X;t} = \frac{2 \quad + 1}{2 \quad + 2} \left( \frac{W_t \quad _t}{Z_t \, \tilde{a}_{X;t}} \right) \tag{B.32}$$

$$N_t \tilde{p}_t = N_{D;t} \tilde{p}_{D;t} + N_{X;t} \tilde{p}_{X;t} \tag{B.33}$$

$$N_t \; \tilde{p}_t = N_{D;t} \tilde{p}_{D;t} + N_{X;t} \tilde{p}_{X;t} \tag{B.34}$$

Variable Markups

$$\widetilde{mu}_{D;t} = \frac{1}{2 \quad + 2} \frac{W_t}{Z_t \tilde{a}_{D;t}} \tag{B.35}$$

$$\widetilde{mu}_{X;t} = \frac{1}{2 \quad + 2} \frac{W_t \quad _t}{Z_t \tilde{a}_{X;t}} \tag{B.36}$$

$$\widetilde{mu}_{D;t} = \frac{1}{2 \quad + 2} \left( \frac{W_t}{Z_t \, \tilde{a}_{D;t}} \right) \tag{B.37}$$

$$\widetilde{mu}_{X;t} = \frac{1}{2 \quad + 2} \left( \frac{W_t \quad _t}{Z_t \, \tilde{a}_{X;t}} \right) \tag{B.38}$$

Bond Market Equilibrium

$$B_{H;t+1} \quad + B_{H;t+1} \quad = 0 \tag{B.39}$$

$$B_{F;t+1} \quad + B_{F;t+1} \quad = 0 \tag{B.40}$$

Labor Market Equilibrium

$$L = \frac{}{2 \; ( \quad + 1)( \quad + 2) P_t W_t} \left( \frac{W_t}{\tilde{a}_{D;t} Z_t} \right)^2 N_{D;t} + \frac{}{2 \; ( \quad + 1)( \quad + 2) W_t} \left( \frac{W_t \quad _t}{\tilde{a}_{X;t} Z_t} \right)^2 N_{X;t} + \frac{N_{E;t} f_{E;t}}{Z_t} = 1 \tag{B.41}$$

$$L \quad = \frac{}{2 \; ( \quad + 1)( \quad + 2) W_t} \left( \frac{W_t}{\tilde{a}_{D;t} Z_t} \right)^2 N_{D;t} + \frac{}{2 \; ( \quad + 1)( \quad + 2) P_t W_t} \left( \frac{W_t \quad _t}{\tilde{a}_{X;t} Z_t} \right)^2 N_{X;t} + \frac{N_{E;t} f_{E;t}}{Z_t} = 1 \tag{B.42}$$

Final Goods Technology

$$F_t = \frac{N \ _t}{2 \ ( \ \ + 1)P_t} \quad \frac{W_t}{Z_t a_{D;t}} \qquad \frac{N_t}{4 \ ( \ \ + 1)( \ \ + 2)P_t^2} \quad \frac{W_t}{Z_t a_{D;t}}^{2}$$

$$\overline{\ }\ \ 2 \quad \frac{N_t}{2 \ ( \ \ + 1)P_t}^{2} \quad \frac{W_t}{Z_t a_{D;t}}^{2} = C_t \qquad \text{(B.43)}$$

$$F_t = \frac{N \ _t}{2 \ ( \ \ + 1)} \quad \frac{W_t}{Z_t a_{D;t}} ! \qquad \frac{N_t}{4 \ ( \ \ + 1)( \ \ + 2)} \quad \frac{W_t}{Z_t a_{D;t}} !^{2}$$

$$! \quad \overline{\ }\ \ 2 \quad \frac{N_t}{2 \ ( \ \ + 1)}^{2} \quad \frac{W_t}{Z_t a_{D;t}} !^{2} = C_t \qquad \text{(B.44)}$$

$N \ _t$

$N_t ec C_{D;t}$

Figure 8: U.S. data: HP ltered trend

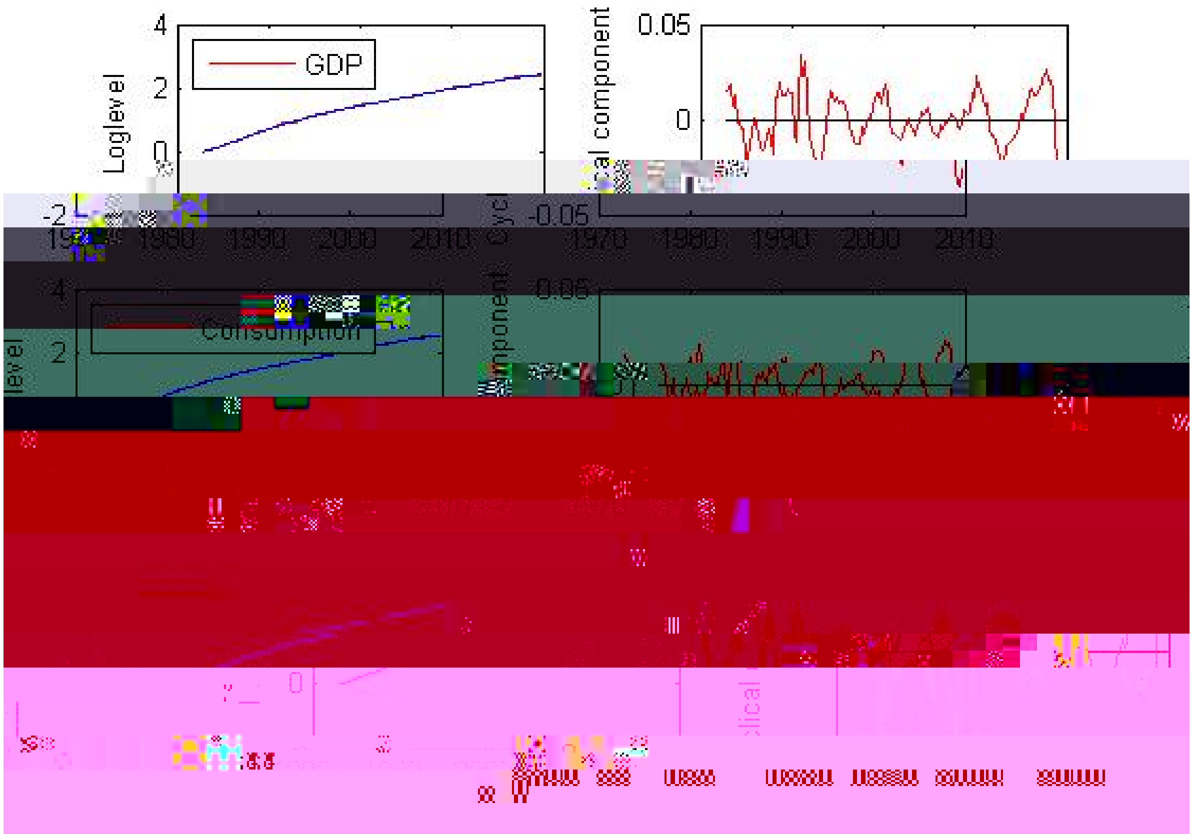


Figure 9: Dynamic Responses to Home Aggregate Productivity Shock

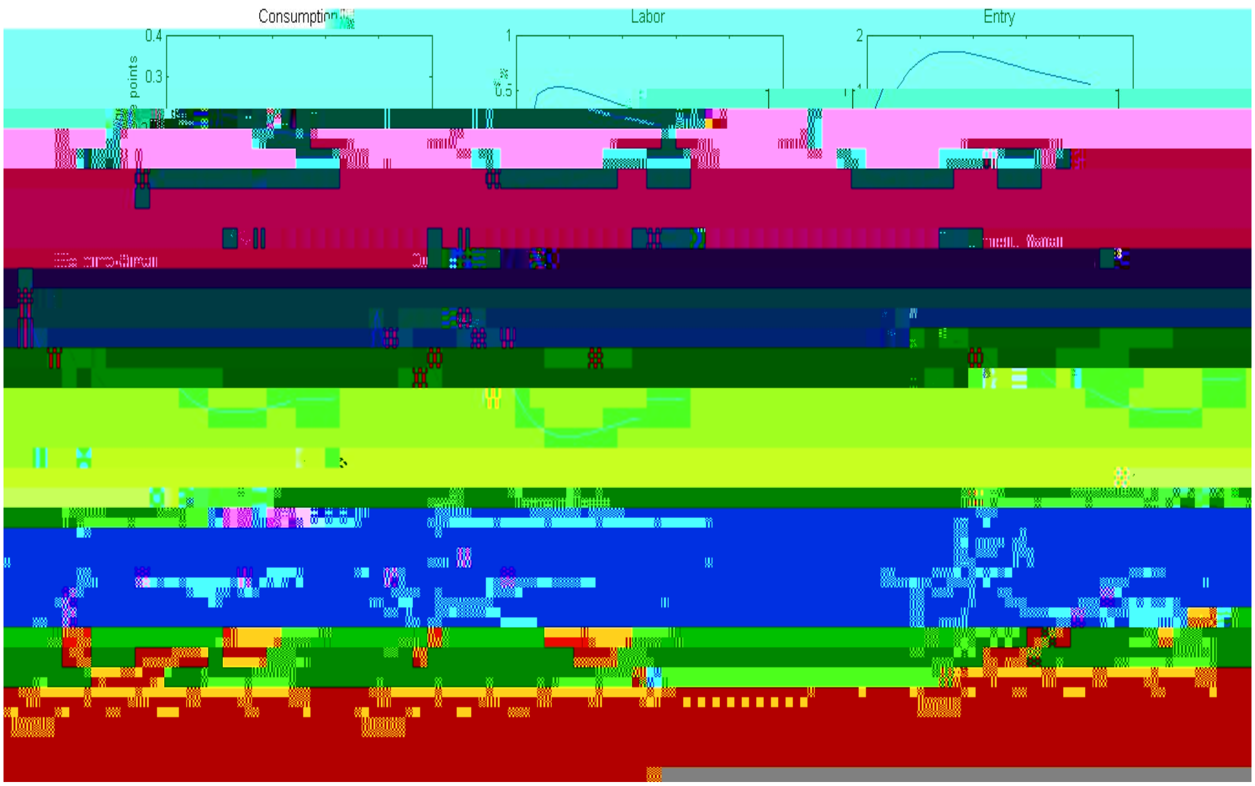


Figure 10: Dynamic Responses to Permanent Increase in $Z_t$


Figure 11: Dynamic Responses to Permanent Decrease in $_t$ and $f_E$