DISCUSSION PAPERS IN ECONOMICS

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Exports, Imports, FDI and Economic Growth

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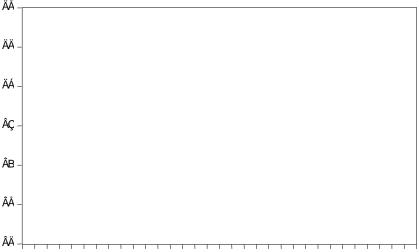
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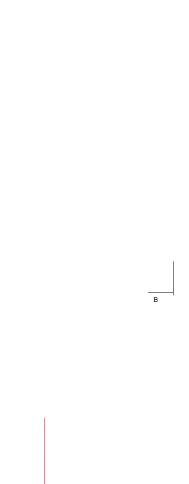
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 $\label{eq:continuous_series} $$ YdJjf ManJj91 [Yffgl Z] jJbJ[l] \ Yll'] -+2 dJnJdJZJ[Ynk] l'] nYkhJ g^u_{H<S}&'akdJkkl'Yfl'] 2 hJj[Jfl [jala[YdnYkhJ g^0-51+O'akkm_]klkl'Yll'] f ne ZJj g^[gafl]_j Ylagf n][lgjkakl'j]]+ $$ YdJjf ManJkl g^u_{H<S}&'akdJkkl'Yfl'] -+2 dJnJdJZJ[YnkJ l'] nYkhJ g^u_{H<S}&'akdJkkl'Yfl'] --2 dJnJdZ[YnkJ l'] nYkhJ g^u_{H<S}&'akdJkkl'Yfl'] nYkhJ g^u_{H<S}&'akdJkkl'Yfl'] --2 dJnJdZ[YnkJ l'] nYkhJ g^u_{H<S}&'akdJkkl'Yfl'] nYkhJ g^u_{H<S}&'a$

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		&. 1 6312'	& +603-5'	&. +4/0'	& -3 0644'
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 $Ggcaf_\ Yl\ OYZd]\ .5)\ l^\]\ ^dn[lnMagfk\ g^B?\ K\ Yj]\]\ phoYaf]\ e\ Yaf\ dj\ Zq\ B?\ K\ Yf\]\ phgjl\ k^\ g[ck+B?\ K\ k^\ g[c\ Y[[gnflk\ ^cj]\ 25-6\#\ Yl\ l^\]\ ^ajkl\ q]\ Yj+Dk\ hjghgjlagf\ af\ l^\]\ nYjaYf[]\ g^B?\ K\ ^\][j]\ Yk]\ k\ gn]j\ lae\]\ Yf\ j]\ Yj+Qphgjl\ k^\ g[c\ Y[[gnflk\ ^cj]\ .1-04\#\ af\ l^\]\ ^ajkl\ q]\ Yj+Qphgjl\ k^\ g[c\ Y[[gnflk\ ^cj]\ .1-04\#\ af\ l^\]\ ^ajkl\ q]\ Yj+Qphgjl\ k^\ g[c)\ o^\ af\ Yk]\ k\ ynj\ lae\]\ Yl\ ynj\ [gflaf\ nynkdl\ ^cj]\ af\ Yj)\ [gflaf\ nynkdl\ ^cj]\ af\ l^\]\ ^ajkl\ q]\ Yj+Qphgjl\ k^\ g[c\ ak\ l^\]\ e\ gkl\ ae\ hgjl\ Yl\ kgnjj\ lae\]\ Znl\ kladl\ Y[[gnflk\ ^cj]\ sdl\ hdYq]\ Zq\]\ phgjl\ k^\ g[c\ af\ [j]\ Yk]\ k\ gn]j\ lae\]\ Yf\ Y[[gnflk\ ^cj]\ s5-83\#\ af\ l^\]\ kapl^\ q]\ Yj+Db\ Y\ alagf)\ l^\ af\ l^\]\ ^ajkl\ q]\ Yj)\ ^Ydk\ lg\ .2+\#\ af\ l^\]\ l^\ al\ phoYaf\ k\ fgl\ ka_f\ a^a[Yll\ l^\]j]\ Yl]\ j+\ A?\ D$

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 $O`] klae Y] \ j] kmdk ^jge \ l`] \ BjYf_]j \ [YmkYddq,=dg[c] pg_]f] alq \ l]kl) \ l`] \ ae \ hmdk] j] khgf k] \ l]kl Yf \ nYjaYf[] \][ge hgkalagf \]e gf kljYl] \li`] phgjl*d] \ _jgo l` `qhgl`] kak+O`]j] ak [YmkYddq g^ae hgjlk gf B? K) Zml l`ak ae hY[l ak o] Yc+R] [Yff gl _an] Yf q [gf[dnkagf YZgml l`] ae hY[l g^ljY,] ddZ]jYdr Ylagf gf][gf ge a[_jgo l` af l`] [Yk] g^F gj] Y+$

 $O`] \ \ a^{\}j]f[]k\ af\]klae\ Y]\ \ j]kmdk\ Z]lo]]f\ Fgj]Y\ Yf\ \ H\ YdYqkaY\ Ykc\ ^gj\]phdYf\ Ylagf\ +\ N][lagf\ 0+l\ o\ adde\ Yh\ l`]\][gf\ ge\ a[\ hgdb[a]k\ g^Fgj]Y\ Yf\ \ H\ YdYqkaY\ gf\ lg\ l`]\ j]kmdk\ g^l`]\ Bj\ Yf\][Ymk\ Ydlq,=dg[c\]pg_]f]alq\ l]kl)l`]\ ae\ hmdk]\ j]khgf\ k]\ ^mf[lagf\ Yf\ n\ YjaYf[]\][ge\ hgkalagf\ +\ N]$

+& 3 XLV:] TJ\$5 YWZL\ TJYX LXO7 bZYLXL/TJX YQ/SP6 TQP\ PXNP] TX 'SP7] ^TWL/POCP]_V] QYW? LYLC] TL LXO= Y\ PL

 $O`ak k][lagf [gf kaklk g^lo g h Yjlk+O`]^ajkl h Yjl]phdYaf k l`]]klae Yl] \ j]kmdk l`jgm_` Yf Ydqkak g^H YdYqkaYf][gf ge a[hgda[q+O`] k][gf \ h Yjl]phdYaf k l`]]klae Yl] \ j]kmd l`jgm_` Yf Ydqkak g^F gj]Yf][gf ge a[hgda[q Yf \ hgaf lk gml l`] \ a^]j]f[]k Z]lo]]f l`] lo g [gmf lja]k+$

+&&? LVLc]TL

H YMqkaYk`a'l]\ 'jge Yf æ hgjl*knZklalnlægf hgdl[q lg Yf]phgjl*gja]fl]\ hgdl[q af . 635 $jaf_al) Yf (gl) je l'g k \& ge g /--0 h+16'+Ajge l'].64-k gf (l') _gn jf e]f l [gf ka]j | l']$]phgjl*gja]fl]\ hgda[q lg Z] l`] e gkl æ hgjlYfl dgf_*l]je hgda[q ^gj \]n]dghaf_ H YdYqkaYkk $\label{eq:local_problem} $ [gfgeq+D] \] \] \] km[[]kkg^1 akhgdq kmhhgjlkeq^af af_l M] phgjlk M] \] Yhgkalan] $$]^][1 gf H YdYqkaYf][gf ge a[_jgo l` + OYZd] . 6 k` go k l` Yl H YdYqkaY46]phgjl k` Yl] g^ B? K $af[j]Y_k$ \ ^jge 1. +1# af . 64- lg . . - \text{# af } / \cdot - 4+O`] Y_n]jY_]]phgjl _jgol` jYl] Yf\ l`] . 6'+Dî l`]][gfge a[Yf\^afYf[aYd[jakak)Y`]Ynq\][j]Yk] af]phgjl_jgol`jYl]d[\ lg Y\][j]Yk] af 1`] B? K_jgo 1` jY]+OYZd] . 6 Ydkg j]hgjlk 1`] B? K]phgjl k`Yj]) 1`]]phgjl _jgo 1` jY]) Yf\ 1`] B? K_jgo1` jYl] g^H YdYqkaY^jge . 64- lg /--4+Agj]pYe hd]) af 1`] . 641v42 gad[jak]k)1`]]phgjl_jgo1`jYl]^]dd^jge .2-6# &641'lg *0# &642')o`a[`hYjldqd]\ lg Y\][j]Yk] af 1`] B? K_jgol^jM] ^jge 5-0# &641' lg - 5# &642'+O'] . 665 < kaYf ^af Yf [aYd[jakak [Ymk]\ l'] $] phgjl _jgol`jM] \; lg \; ^Ydd ^jge \; 6 + \# \; \&663' \; lg \; - 2 \# \; \&665') \; o \; a[\; d] \setminus \; lg \; Y \setminus][j] \; Yk] \; af \; B? \; K$ _jgo l` jYj] ^jge .-# &663' lg *4+1# &665'+O`]k] ^Y[lk Yj] [gf kakl]fl o al` gf] *o Yq [Ymk Ydalq g^]phgjlk gf][gf ge a[_jgo l` +

Og hjge gl]]phgjlk) l`] H XtNqkaY_gn]jfe]fl]p]e hl]\ Yf æ hgjl lYp gf afl]je]\aM] af hmlw [YhalYd_gg\ kw l`Yl o]j] mk]\ ^gj]phgjl hjg\ m[lægf gj æ hgjl*kmZklalmlægf af\ mklja] k+O`]j]^gj]) af H XtNqkaY) æ hgjlk g^ afl]je]\aM] _gg\ k Yf\ [YhalYd_gg\ k hjge gl]\]phgjl e Yf mfY[lmj] Yf\ \ge]kla[hjg\ m[lægf + Df gl`]j o gj\ k) æ hgjlk `Y\ Y [YhkYd]j\dMæf k` ah o al`]phgjlk Yf\ B? K+O`]]phgjl nYdn] g^ H YdNqkaYaf[j]Yk]\ ^jge . +4 Zadbgf P N? af . 64- lg . . 1 Zadbgf P N? af /--1 &OYZd] . 6') Yf\ hjgna\]\ Y ^gj]a_f]p[`Yf_] kgmj[] ^gj j]æ hgjlaf_afl]je]\aM] af hml) [YhalYd_gg\ k) Yf\ [gf kme hlæf _gg\ k+J f] [Yf k]] l`Yl'] H YdNqkaYf æ hgjl

 $nYth] Ytkg af[j]Yk] \ \ \, \'ige . - B Zattlef PN? af . 64- lg ... 6 Zattlef PN? af /--1 & gjd, ZYfc ? YYk]1*R? D+Df /--1)1`] af1]je]\ aYl] _gg\ k Yf\]i mahe]f1 ^gj]phgjl e Yf mYy[mjaf_`Y\ fgl q]1 Z]]f hjg\ m[]\ Zq \ ge]kla(af\ mklja]k) Yf\ 1` mk1`] [gmfljq klatdl`Y\ lg ae hgjl+O`]j]^gj]) Yf af[j]Yk] af B? K Ykck ^gj af[j]Yk] af ae hgjlk g^ af1]je]\ aYl] _gg\ k Yf\ [YhalYd_gg\ k+O`]k] ^Y[lk Yj] [gf kakl]f1 o al` 1`]]klae Yj]\ j]kmdlk ^jge Bj Yf_]j > YmkYthla]k) 1`Yl]phgjlk Yf\ ae hgjlk Yk o]tdYk ae hgjlk Yf\ B? K af H YtYqkaY`Yh] lo g*o Yq [YmkYthla]k+$

ELMP) 127 bZY\ ^DSL\ P YX 9 6 B LXO7 bZY\ ^9 \ Ya ^S CL/P YQ? LYLc] TL\$) 1/(%((/ '! #

J PL\	7 bZY\ ^	7 bZY\ ^	96B	J PL\	7 bZY\ ^	7 bZY\ ^	96B
	DSL\ P YX	9 \ Ya 'S	9 \ Ya 'S		DSL\ PYX	9 \ Ya 'S	9 \ Ya 'S
	96B	CL/P	CL/P		96B	CL/P	CL/P
TM 64-	. 0-₿	. 6-₿	5-0	TM 656	05	*1	3-4
TM 64.	. 2	/ . -4	5≠	TM 66-	/ 5	1-2	6 ⊬
TM 64/	. 6+1	04⊬	1-2	TM 66.	/ 3-0	+	6-1
TM 640	/ 5 -4	23	./+	TM 66/	/ 3-3	./₩	2-6
TM 641	/ 3-4	*. +6	4+⁄	TM 660	/ 3-2	./₩	3+
TM 642	/ 3-16	. 5-4	2-16	TM 661	/ 3-3	. 3-0	5-2
TM 643	0-	06-2	B	TM 662	/ 5-5	/ 1-1	6+

TM 644 0-4

ELMP*(2ESP5 YWZYXPX^YQ? LYLc]TLX7 bZY\^] YX? LX_QN_\TXR\$) 1/(%11-'! #

			<u> </u>	17 . (7.		
;XO_]^ T]) 1. 0) 1/ +) 10() 10-)11() 11-
Agg∖	. 4-2	. 6-₿	2-4	3≠	0-5	. 45
$=]n]jY_]kYf \setminus lgZY[g$	-+6	/ -16	- - -	- ≠	- ≠	-+0
O]plad])[dgl`af_Yf\^gglo]Yj	. +1	3÷	2	1 6	5-5	1-3
R gg\	0+1	6-4	2-4	0+	0+1	1+1
>`]e a[Ydk	0+	1-1				

 $\begin{tabular}{ll} M$?) Yk o] dtlYk af ljY \af_ le Yjc] laf_ Yf \dg[YdZjYf \] n] dghe] fl+<fa? Dhjgb] [l o Yk Yo Yj \] hagf]]j klYlnk af al kYlakfa] \lambda [j] [jal]jaY7 0-# gj e gj] nYdn] Y \] \lambda [Y] \lambda [Y] \dg Yd e Yf Y_]jaYd l] [fal Yd) gj kmh]jnakgjq h]jkgff] dy Yf \Y [gfljaZmlagf dln] dlg H YdYqkaYxk af \mkljaYd kljm[lmj] & ge g) /--0 \h+.--'+A? Dak hj]^jj] \af \a_\alpha \] l] [fgdg_q af \mklja]k km[\Yk [ge hml]jk) G>? k & dhi ma \[[jqklYd \akhdYqk') e] \a[Yd] i mahe]fl) Zag*l][\fgdg_q af \mklja]k km[\Yk [ge hml]jk) G>? k & dhi ma \[[jqklYd \akhdYqk') e] \a[Yd] i mahe]fl) Zag*l][\fgdg_q y Ymlge Ylagf]i mahe]fl) Y \n Yf[] \e Yl]jaYd \] d[[ljgfa[k) kg^lo Yj]) Ydl]jf Ylan] [f]j_q Yf \YjjgkhY[]+D kme e Yjq) A? Daf[j]Yk] \hjg \m[lanalq 1 \jgm_ \hjge glaf_ 1][\fgdg_q Yf \affgnYlagf) o \alpha[\frac{1}{3} \$

$+&& = Y_1PL$

 $\label{eq:final_state} Fgj]Yak Yke Yktlj Yj]Y)Znl`Yk YdYj_]j hghmtMagf &15-60 e attlegf)o`a[`ak f]Yjdq lo a[] l`Yl g^ H YdYqkaY Yf\ hggj]j f YmjYd j]kgmj[]k l`Yf H YdYqkaY & YY kgmj[]7 P-N+?]hYjle]fl'+O`]j]^gj])l`] Fgj]Yf _gn]jfe]fl[`gk] l`]aj Bjgo l`*D\ mkljq J mlo Yj*J ja]fl]\ kljYl]_q &B D' & Qf_)/--0)h+.0'+$

 $<11^{\circ}] Z]_{aff} af_g^{\circ}]]phgjl*gja]fl] \land pdd(q) Fgj] Yo Yk \ a^{\circ}]jfl ^{\circ}ge \ H YdYqkaY+R ^{\circ}a] af 1^{\circ}] . 63-k)]phgjlk g^ hjæ Yjq _gg\k \ ge af Y]\ 1^{\circ}]]phgjl _gg\k g^ H YdYqkaY)]phgjlk g^ e Yf mYl[mjaf__gg\k Y[[gmfl]\ ^gj lo g*l^aj\k g^1^]]glYdFgj] Yf]phgjl _gg\k +GYZgj*afl]f kan] e Yf mYl[mjaf__gg\k Y[[gmfl]\ ^gj 4-# g^lglYd]phgjl*e Yf mYl[mjaf__gg\k &^ g) . 661) h+ 14'+ R ^al] H YdYqkaY^ Yk [gflafm]\ lg [gfka\]j]phgjl gja]f lYlagf Yk alk d] Yl af_ hgdb[q) ^jge 1^{\circ}] . 63-k lg 1^{\circ}] hj]k]fl) Fgj]Yj]Ydr]\ 1^{\circ}Y e Ypæ ar af_]phgjl k o Yk fgl Ydb Yqk Y_gg\ hgdb[q) Yf\ e Yl Yk bnkle]fl af][gfge a[kljYl]_q+C]f[]) 1^{\circ}] Fgj]Yf]phgjl k Yj] g^ B? K af[j]Yk]\ ^jge . 0-\text{8} af . 64- lg 0/+# af 1^{\o}] . 65- k &OYZd] /.') Yf\ o Yk 1^{\o}f c]hl Yjgmf\ 0-# v1-# mh lg 1^{\o}f hj]k]fl+Cgo]n]j) 1^{\o}f H YdYqkaYf]phgjl k Yj] g^ B? K af[j]Yk]\ [gflaf mgmkdp ^jge 1. +l# af . 64- lg ... -## af /--4 Yf\ Yl Y a_ YhjjY_] d|n]dg^41-2# &OYZd] /.'+$

 $Naf[\] \ '\] \ .63-k) \ Fgj]Y \ 'Yk \ j] \ 'Kar] \ '\ 'Yh \ |\ 'Nah \ 'Yh \ |\ 'Yh \ '$

)1/)%-) 1/. %)() 10) %-) 10. % (11) %+
= Y \ PL	16	- +1	2	. +0	- - B
? LVc]TL	. 2⊬	2	5	2	/ 1-3

 $G(b_PR3IB7H58"NPVRQV>Z\backslash\%)), \%'+\#$

 $=]^gj] l^3 - f Yf [aYd [jakak) l^3] ggd g^A? Do Yk f gl j] [g_f ar] \ Zq l^3] F gj] Yf _gn] f le f le Yn] f g af [] f lan] k lg Yllj Yl A? D \ m] lg ^] Yj l^Y Y ^gj] a_f [ge h Yf q o gmd \ ge af Yl] l^3] e Yjc] l+J f l^3] gl^3] Yf \ Yk e] f lagf] \ YZgn] l^3] \ ge] kla(^aje k \ a\ f gl ^ Yn) Yf q af [] f lan] lg$

, 5A@5>FD;A@D

O`ak [`Yhl]j Yhhdı]k l`] ^gmj*nYjaYZd] Q<Meg\]d) o`a(`ak [gfkljm[l]\ ^jge ^gmj]f\g_]fgmknYjaYZd]kw l`] dg_Yjal`e g^j]YdB? K) l`] dg_Yjal`e g^j]Ydlphgjlk) l`] dg_Yjal`e g^j]Ydlæhgjlk Yf\ l`] dg_Yjal`e g^A? Dv af gj\]j lg gZk]jn] l`] afl]_jYl]\ j]dMagfk`ah Z]lo]]f ljY\]) A? DYf\][gfge af _jgo l` ^gj H YdYqkaY Yf\ Fgj]Y\mjaf_ l`] læ] h]jag\ ^jge . 64- lg /--1 &H YdYqkaY Yf\ ^jge . 643 lg /--4 &Fgj]Y+

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C787C7@57D

- | f lagm) K+Yf\ <+Njjd| lak+/---+J mlhml Bjgo l`Yf\ QYjaYZaddq g^@phgjl Yf\ To hgjl Bjgo l`7
 To ljif Ylagf Yd @na\]f[] ^jge BjYf_]j > YmkYddq O] klk+HUR8RCRX / VT9P [\ ZVR LLLJ ==XT . 1. VT30+
- <d_mM(ad) H +O+Yf\ Q+J jlk+/--0+Dfo Yj\ Agj]a_f ? aj][1 Dfn]kle]f1 Yf\ De hgjlk af NhYaf+ \neq aR_J NaW | NY9 P_i\ Z VP > b_NY. 47. 6v05+
- $< kYm^* < bYq]) E+Yf \ ? +> `Ycj YZgjlq+. 666+@phgjl*G] \ Bjgo l` Yf \ De hgjl > ge hj]kkagf 7 Anjl`]j Oae] Njja]k @na\]f[] 'jge <math>G? > k+5b`a_NW9P \ [\ ZVP' DN]R_` 057. 31v. 42+$
- = Y Yjme k' Y) < +U+Yf\ N+MYk' a\ +. 666+@phgjlk) De hgjlk Yf\ @[gfge a[Bjgo l' af H YdYqkaY7 @e haja[Yd@na\]f[] = Yk]\ gf H mdan YjaYl] Oae | N|ja|k+5`W| $9P_1/ZVP > b_1/NY$. 07056v1-3+
- $= Y e Yfa^*J kcgg]]) H + Yf \ C + E+M]] + .664 + <j] De hgjlk Yf \ @phgjlk g^F gj] Y > gafl]_jY] \ ;$ $= dR \int NdV [NY9P_i \ Z + D JN. .7. 6v. .. +$
- O+) R +AYf_) R +Gam) Yf\ O+C]fjq+/---+@phgjlk De hgjlk Yf\ De [ge] af OYao Yf7<f @pYe af Ylagf g^1`] @phgjl*G]\ Bjgo 1` Cqhg1`] kak+= $\frac{1}{2} \frac{1}{2} \frac{1}{2$
- >`g) N+. $661+HUR8f[NZVP` \ S? \ RV[9P\ [\ ZVP8RcRX]\]ZR[a+R Yk`af_lgf? > 7Diklalml] ^gj$ Dil] jf Ylagf Yd @f ge a[k+
- $@f \setminus jk$ R +/--0+5]] $XRQ9P \setminus [XRa_VPHZRGR_VR+CgZgc]f$ I E/R adjq+
- @k^Y Yf a) C+N+. 66. +@phgjlk) De hgjlk Yf\ @[gf ge a[Bjgo l` af N]e a*D\ mkljadar]\ > gmf lja] k+ $\Rightarrow b \int M \setminus S8 RcRY [ZR] a9P [\setminus ZVP] 02760v...3+$
- $Anf_) C \not= B \not= +NYo `f] q) R \not= +Gg) Yf \ K+S aYf_+. 661+@phgjlk) De hgjlk Yf \ Df \ mkljaYd Kjg \ mf lagf 7 @na\]f[] ^jge < \nYf[] \ Yf \ I]o dq Df \ mkljaYdar af_ > gmf lja] k+<math>\neq aR \int NdV [NY + QR \int NdV] Nf \ P = NYS + P = NYS +$
- CkaYg) A+N+O+Yf\ H \not >+R +CkaYg+/--3+A? D @phgjlk) Yf\ B? K af @Ykl Yf\ Ngml`] Ykl < kaYw KYf]d? YlYn]jknk Oae] N]ja]k > YnkYddq < f Ydqk]k+ \Rightarrow b $\int MY S5$ `W $\int P \{ |X|^2 \cdot 47.-5/v..-3+ MY S5$
- Cmkl]\) N+. 66/+0`] @e]j_af_P+N+>mjj]f1 < [[gmf1?]^a[al af 1`] . $65-k7Y>gaf1]_j$ Magf < f Ythkak+HUR FRc VRd \ $S9P_i$ \ Z VP' N/Q GaNdV dP' 417. 26v. 33+

IDH A + & - - 6' +

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