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Search, Heterogeneity, and Optimal Income Taxation

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WORKING PAPER

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November 9, 2009

Abstract

D . N , M (1982), H (1990) A (1999) . H , (2001) , . I .

. I . , . . I

2 Model

(H) (L)

l_k, $k = H; L$, q_m, $m = H; L$.

$y_{km} > 0$,

$y_{Hm} > y_{Lm}$.

$\in 0; 1$,

$c_w(\ _k)$,

$c_w(0) = 0$, $c'_w(0) = 0$, $\delta_k \to 1$ $c'_w(\ _k) = +\infty$.

v_m, $c_\pi(v_m)$

O … (…). A … , … . A … , … , … .

2.1 The matching technology

… , … , … , … , … , … , … , … , … , … , … .

… $_k$ $_kl_k$. … , … , … , 2255( )-256-282(390( )23)-28 5.-33

, 1 – ff
. A M P (1999) ,
. H , P (1996) Bl D -

2.2 Output sharing

2.4 Private expected utility functions

$v_H q_H = \quad _m q_m$.

k

$$U_k = -c_w(\quad _k) + \quad _k \quad M(\quad) \quad \frac{v_H q_H}{_m v_m q_m} \quad _{kH} y_{kH} + \frac{v_L q_L}{_m v_m q_m} \quad _{kL} y_{kL} \quad + (1 - M(\quad))0 \quad + (1 - \quad _k)0$$

$$U_k = -c_w(\quad _k) + \quad _k M(\quad) E_{(m)} \quad _{km} y_{km}; \tag{5}$$

$E_{(m)}$

. A , , $c(\quad)$, . O , ,

$M(\quad)$,

. I ,

$1 - M(\quad)$, , ,

. I ,

(

[illegible]

3 Optimal search intensity and market ine ciencies

[illegible]

3.1 Social Optimum

A [illegible]:

$$W = \max_{\delta, v} \quad l_k U^k + \quad q_m V^m$$

$$\ldots_k \geq 0; \quad v_m \geq 0.$$

[illegible] (1), (2), (5), [illegible] (6), [illegible] -91.21 [illegible]

$$E_{(m)}y_{km} - (1 - \quad)E_{(k)}E_{(m)}y_{km} = \quad E_{(k)}E_{(m)}y_{km} + E_{(m)}y_{km} - E_{(k)}E_{(m)}y_{km}:$$

A ... (...) ... ($E_{(k)}E_{(m)}y_{km}$), ... ff ... (... (...) ...) ... , $E_{(m)}y_{km}$, ... , $E_{(k)}E_{(m)}y_{km}$. ... , ... (...) ff ... .

... :

... 1 . I ...

$$\begin{array}{ll} c'_\pi(\bar{v}_H) = \frac{M(\bar{\ })}{\bar{\ }} E_{(k)}y_{kH} - \quad E_{(k)}E_{(m)}y_{km} & \\ c'_\pi(\bar{v}_L) = \frac{M(\bar{\ })}{\bar{\ }} E_{(k)}y_{kL} - \quad E_{(k)}E_{(m)}y_{km} & \end{array} \quad \left| \begin{array}{l} \bar{\ } \lesseqgtr 1; \\ \bar{v}_H > 0;\ \bar{v}_L > 0 \end{array} \right. ; \qquad (13)$$

$$\begin{array}{ll} c'_\pi(\bar{v}_H) = \frac{M(\ )}{\ } E_{(k)}y_{kH} - \quad E_{(k)}E_{(m)}y_{km} & \\ c'_\pi(0) \geq \frac{M(\ )}{\ } E_{(k)}y_{kL} - \quad E_{(k)}E_{(m)}y_{km} & \end{array} \quad \left| \begin{array}{l} \bar{\ } \lesseqgtr 1; \\ \bar{v}_H > 0;\ \bar{v}_L = 0 \end{array} \right. : \qquad (14)$$

3.2 Decentralized equilibrium

A ... k ...

$$\max_{\delta_k} \quad U_k = -c_w(\ _k) + \ _k M(\ )E_{(m)}\ _{km}y_{km} \qquad (15)$$
$$\text{s. t.} \quad _k \geq 0;$$

... $C_k = c_w(\ _k)$ ... , ... $B_k = \ _k M(\ )E_{(m)}\ _{km}y_{km}$ ... fi ... .

K [12]

$$-c'_w(\ _k) + M(\ )E_{(m)}\ _{km}y_{km} \le 0$$

$$_k \ge 0 \qquad (16)$$

$$(-c'_w(\ _k) + M(\ )E_{(m)}\ _{km}y_{km})\ _k = 0;$$

$_k$, . $M(\ )$, . F ,

ff ,

ff .

I -

. $_{LL} \neq 0$ ($_{LH} = 0$), $y_{km} > 0$,

fi (06)2 158.730 ( ) -395. 4.9812163(0),)-373( )-333(( )-350(5. 4

[13] A high type worker, for example, has to search less intensively than a low type worker to achieve the same expected income as a low type worker.

. D l ll K (1992), ll B B (2002),

ff ll ff ;

fi [14]. fi

. O fi

.

4. *I*

$$c'_w(\ \ _k) = M(\ \ )(1 - \ \ ^w_k)w_k \qquad \lesseqgtr 1; \quad ; \quad _k > 0;\ v_m > 0$$
$$c'_\pi(v_m) = \frac{M(\ \ )}{}(1 - \ \ ^\pi_m)\ \ _m \tag{22}$$

$$c'_w(0) \geq M(\ \ )(1 - \ \ ^w_L)w_L \qquad \lesseqgtr 1; \quad : \quad _L = 0;\ v_L = 0$$
$$c'_\pi(0) \geq \frac{M(\ \ )}{}(1 - \ \ ^\pi_L)\ \ _L \tag{23}$$

4.1 Characterizing externalities through Pigou taxes

, P $(\check{\ }^w_H; \check{\ }^w_L)$, P $(\check{\ }^\pi_H; \check{\ }^\pi_L)$. L ( ) $\check{R}$, P , , LS:

$$\check{R} = (\ \ _k\ \ l)M(\ \ )\ \ \frac{_H l_H}{_k\ \ l}\check{\ }^w_H w_H + \frac{_L l_L}{_k\ \ l}\check{\ }^w_L w_L + \frac{v_H q_H}{_m vq}\check{\ }^\pi_H\ \ _H + \frac{v_L q_L}{_m vq}\check{\ }^\pi_L\ \ _L$$

$$0 = \check{R} - \ \ _k l_k + \ \ _m l_m\ \ LS;$$

($_k$ $l)M(\ \ ) = N$.

k

$$U_k = -c_w\ \ \frac{z^w_k}{M(\ \ )\,w_k}\ \ + LS + (1 - \check{\ }^w_k)z^w_k \tag{24}$$

, LS, ff

. fi ,

c'_w

(P 5)

. CR , ;

, . - , P

, .

- , , ff . I

fi , fi , L ,

.

N , ff $(g = 0)$. , . I

$y_{LL} > 0$, [illegible]

[illegible]

4.2 Optimal income taxes with positive government revenue

[illegible] R, [illegible][17]. [illegible]

$$W = \sum_k l_k U^k + \sum_m q_m V^m ;$$

[illegible]

[illegible] (1), (2), (5), [illegible] (6), [illegible] $w_k = E_{(m)} \; {}_{km} y_{km}$, $z^w_k = \; {}_k M(\;) w_k$, ${}_m = E_{(k)}(1 - \; {}_{km}) y_{km}$, [illegible] $z^\pi_m = v_m \frac{M(\theta)}{\theta} \; {}_m$ [illegible] 4, [illegible]

$$W = \sum_k l_k \left(-c_w \frac{z^w_k}{M(\;) w_k} \right) + \sum_m q_m \left(-c_\pi \frac{z^\pi_m}{\frac{M(\theta)}{\theta} \; {}_m} \right) + (\;{}_k\; l) M(\;) \; E_{(k)} E_{(m)} y_{km};$$

[illegible] $(\;{}_k\; l)M(\;) = N$ [illegible].

[illegible] R, [illegible]

$$R \leq (\;{}_k\; l) M(\;) \left(\frac{{}_H l_H}{{}_k\; l} \; {}^w_H w_H + \frac{{}_L l_L}{{}_k\; l} \; {}^w_L w_L + \frac{v_H q_H}{{}_m vq} \; {}^\pi_H \; {}_H + \frac{v_L q_L}{{}_m vq} \; {}^\pi_L \; {}_L \right); \tag{30}$$

[illegible] $(\;{}_k\; l)M(\;) = M$ [illegible] (30), [illegible]

[17] The public good, even if valued by consumers, does not affect their choice on search intensity.

$$W = \sum_k l_k \left(-c_w \frac{z_k^w}{M(\ )w_k} \right) + \sum_m q_m \left(-c_\pi \frac{z_m^\pi}{\frac{M(\theta)}{\theta}\ _m} \right)$$

$$+(\ _k\ l)M(\ )\ \frac{_H l_H}{_k\ l}(1-\ _H^w)w_H + \frac{_L l_L}{_k\ l}(1-\ _L^w)w_L$$

$$+ \frac{v_H q_H}{_m vq}(1-\ _H^\pi)\ _H + \frac{v_L q_L}{_m vq}(1-\ _L^\pi)\ _L + R:$$

fi (22) (C (1978), D (1998)).

H

L 7 A . E , fi (22)

$$\max_{\tau_k^w, \tau_m^\pi} W = \sum_k l_k \left(-c_w \frac{z_k^w}{M(\ )w_k} \right) + \sum_m q_m \left(-c_\pi \frac{z_m^\pi}{M(\theta)} \right)$$

P 8 A .

$''^w_k = 0$ $''^\pi_m = 0$. I . I

(33).

H , ff

P 5 A A). H ,

I $''^\pi_m = \infty$,

$''^\pi_m = \infty$,

4.2.2

I

1

/
( ),
(D (1998)). fi
' (P 11),
, .
P 11 ( )

12.

$$i)\ \partial \frac{w_H}{w_L} = \partial \frac{E_{(m)\ \ Hm}}{E_{(m)\ \ Lm}} < 0 \tag{36}$$

$$ii)\ \partial \frac{w}{\pi} = \partial(\ \ ) < 0 \tag{37}$$

P 11 A . B (5). ( ), .

P 12 . B B (2002) .

5 Conclusion

, . I

[illegible]

[illegible] (1[illegible]) "H[illegible] Effi[illegible] F[illegible]," *I[illegible] E[illegible]*, 40, 827-849.

[illegible] (2002) "O[illegible] L[illegible]," *J[illegible] E[illegible]*, 85, 53-97.

[illegible] (1[illegible]4) "Eff[illegible] B[illegible]fi[illegible] F[illegible] P[illegible]."

[illegible] (1[illegible]) "M[illegible] Cl[illegible]," *J[illegible] E[illegible]*, 112, 141-168.

[illegible] (1[illegible]) "L[illegible] P[illegible] F[illegible]: M[illegible] E[illegible]," *E[illegible] J[illegible]*, 109, 307-334.

[illegible] (1[illegible]) "O[illegible] R[illegible]: M[illegible] R[illegible]," *A[illegible] E[illegible]*, 68, 756-768.

[illegible] (1[illegible]1) "M[illegible] C[illegible], F[illegible] Effi[illegible]," *J[illegible] E[illegible]*, .89, 798-812.

[illegible] (1[illegible]2) "A[illegible] D[illegible] M[illegible] E[illegible]," *J[illegible] E[illegible]*, .90, 881-894.

[illegible] (1[illegible]2) "[illegible] D[illegible] Effi[illegible] E[illegible]," *E[illegible]*, .49, 217-227.

[illegible] (1[illegible]) "O[illegible] I[illegible]: A[illegible] E[illegible] P[illegible] O[illegible] M[illegible] R[illegible]," *A[illegible] E[illegible]*, .88, 83-95.

[illegible] (1[illegible]) "[illegible] P[illegible]," *E[illegible] J[illegible]*, 7, 46-70.

[illegible] (1[illegible]3) "O[illegible] O[illegible] P[illegible] I[illegible]," *J[illegible] E[illegible]*, 2, 357-376.

[illegible] (1[illegible]) "H[illegible] D[illegible] P[illegible] Aff[illegible] E[illegible]? C[illegible] O[illegible] E[illegible]," M[illegible] P[illegible].

. (1 0) "Effi I ," J E , 51, 8999.

., . (1 5) "P , , ," P , D E , .

, . . (1 0) "O Effi M R M ," E , 57, 279-298.

, . (1) " , F , ," E , 32, D E , .

, ., . (1) " P G E P M B ," L E , 3, 6580.

., . (1) "L R , G J B J ," E , C , P .

, ., . (1 3) "

(1) "L M D Eff ," J E , 33, 457495.

(2000) "A M ," E , 68, 343-369.

(2001) "M , , H " A , 1(2001), I . 1, A . 5.

. (1) "P F E P ?," E , 25, 223264.

. (1) "O P F L M ," L E , 6, 435452.

. (1) "P Effi O N N E ," I : H E , .2.

. (1) " - P Effi ," J E , 6, 239262.

. (1) " , P P D ," I F , 6, 239262.

, . (1) " P F E ," I : , .B. (E .), F C ., M M , L .

Appendices:

A Proofs of the main results

Proof of Corollary 3.

F , . - , $_1 \neq$. I $_1 >$, $_H(_1) > {}_H()$, $_L(_1) > {}_L()$, $v_H(_1) < v_H()$, $v_L(_1) < v_L()$. H

)$_1$)

[illegible] CR [illegible]. F [illegible] 1 − =

$$\check{R} = N \left[\begin{array}{l} \frac{\delta_H l_H}{k\,\delta l} (1 - \;) E_{(k)} E_{(m)} y_{km} - E_{(m)} (1 - \; {}_{Hm}) y_{Hm} \\ + \frac{\delta_L l_L}{k\,\delta l} (1 - \;) E_{(k)} E_{(m)} y_{km} - E_{(m)} (1 - \; {}_{Lm}) y_{Lm} \\ + \frac{v_H q_H}{m\, vq} (1 - (1 - \;)) E_{(k)} E_{(m)} y_{km} - E_{(k)} \; {}_{kH} y_{kH} \\ + \frac{v_L q_L}{m\, vq} (1 - (1 - \;)) E_{(k)} E_{(m)} y_{km} - E_{(k)} \; {}_{kL} y_{kL} \end{array} \right]$$

$$= N \left[\begin{array}{l} \frac{\delta_H l_H}{k\,\delta l} (1 - \;) E_{(k)} E_{(m)} y_{km} - E_{(m)} (1 - \; {}_{Hm}) y_{Hm} \\ + \frac{\delta_L l_L}{k\,\delta l} (1 - \;) E_{(k)} E_{(m)} y_{km} - E_{(m)} (1 - \; {}_{Lm}) y_{Lm} \\ + \frac{v_H q_H}{m\, vq} (1 - \;) E_{(k)} E_{(m)} y_{km} - E_{(k)} \; {}_{kH} y_{kH} \\ + \frac{v_L q_L}{m\, vq} (1 - \;) E_{(k)} E_{(m)} y_{km} - E_{(k)} \; {}_{kL} y_{kL} \end{array} \right]$$

$$\check{R} = N \; 1 - (\; + \;)$$

[illegible]. I [illegible] ([illegible] L [illegible]).□

Proof of Lemma 7.

I [illegible] k

$$U_k = \; - c_w (\; {}_k) + \; {}_k M(\;)(1 - \; {}^w_k) w_k$$
$$= \; - c_w \; \frac{z^w_k}{M(\;)\, w_k} \; + (1 - \; {}^w_k) z^w_k$$

[illegible] w_k

$$\frac{\partial U_k}{\partial w_k} = -c'_w \frac{1}{M(\;)\, w_k} \frac{\partial z^w_k}{\partial w_k} - c'_w \frac{z^w_k}{M(\;)} \; -\frac{1}{w_k^2} \; + \frac{\partial z^w_k}{\partial w_k}(1 - \; {}^w_k) = c'_w \frac{z^w_k}{M(\;)\, w_k} > 0;$$

[illegible] E [illegible]. □

Proof of Proposition 8.

$w_k = E_{(m)} w_{km} = E_{(m)} \; {}_{km} y_{km}$

$k = H; L;$ $z^w_k = \; {}_k M(\;) w_k$

$k = H; L;$ ${}_m = E_{(k)} \; {}_{km} = E_{(k)}(1 - \; {}_{km}) y_{km}$

$m = H; L;$ $z^\pi_m = v_m \; {}^M$

Z

k π π

. F fi , , ,

$$\frac{dz^w_H}{z^w_H} \quad \frac{1}{''^w_H} + (1 - \quad) \frac{{}_H l_H}{k \quad l} \quad = (1 - \quad) \quad E_{(m)} \quad \frac{dz^\pi_m}{z^\pi_m} \quad - \frac{{}_L l_L}{k \quad l} \quad \frac{dz^w_L}{z^w_L} \quad - \frac{d \quad {}^w_H}{1 - \quad {}^\pi} \qquad ($$

L

[illegible]

$$\frac{dz_H^\pi}{z_H^\pi}\frac{1}{"_H^\pi} = \frac{E_{(k)}\Big(\frac{dz_k^w}{z_k^w} \quad + \frac{d\tau_L^\pi}{1-\tau_L^\pi} \quad \frac{v_L q_L}{{}_m vq}"_L^\pi - \frac{d\tau_H^\pi}{1-\tau_H^\pi}\Big(1 + \quad \frac{v_L q_L}{{}_m vq}"_L^\pi}{\Delta_2} \tag{45}$$

$$\frac{dz_L^\pi}{z_L^\pi}\frac{1}{"_L^\pi} = \frac{E_{(k)}\Big(\frac{dz_k^w}{z_k^w} \quad - \frac{d\tau_L^\pi}{1-\tau_L^\pi}\Big(1 + \quad \frac{v_H q_H}{{}_m vq}"_H^\pi \quad + \frac{d\tau_H^\pi}{1-\tau_H^\pi} \quad \frac{v_H q_H}{{}_m vq}"_H^\pi}{\Delta_2}; \tag{46}$$

[illegible] $\Delta_2 = 1 + \quad E_{(m)}"_m^\pi$. A [illegible] (45) [illegible] (46) [illegible]

$$E_{(m)} \quad \frac{dz_m^\pi}{z_m^\pi} \quad = \frac{E_{(k)}\Big(\frac{dz_k^w}{z_k^w} \quad E_{(m)}"_m^\pi - E_{(m)}\Big("_m^\pi\frac{d\tau_m^\pi}{1-\tau_m^\pi}}{\Delta_2}; \tag{47}$$

[illegible] (43) [illegible] (44) [illegible]

$$E_{(k)} \quad \frac{dz_k^w}{z_k^w} \quad = \frac{(1- \quad)E_{(m)}\Big(\frac{dz_m^\pi}{z_m^\pi} \quad E_{(k)}"_k^w - E_{(k)}\Big("_k^w\frac{d\tau_k^w}{1-\tau_k^w}}{\Delta_1}: \tag{48}$$

F [illegible] (47) [illegible] (48) [illegible] $E_{(m)}\Big(\frac{dz_m^\pi}{z_m^\pi}$ [illegible] $E_{(k)}\Big(\frac{dz_k^w}{z_k^w}$ [illegible]

[illegible]

$$E_{(m)} \quad \frac{dz_m^\pi}{z_m^\pi} \quad = -\frac{(\Delta_2 - 1)E_{(k)}\Big("_k^w\frac{d\tau_k^w}{1-\tau_k^w} \quad + \Delta_1 E_{(m)}\Big("_m^\pi\frac{d\tau_m^\pi}{1-\tau_m^\pi}}{\Delta_1 + \Delta_2 - 1} \tag{49}$$

$$E_{(k)} \quad \frac{dz_k^w}{z_k^w} \quad = -\frac{\Delta_2 E_{(k)}\Big("_k^w\frac{d\tau_k^w}{1-\tau_k^w} \quad + (\Delta_1 - 1)E_{(m)}\Big("_m^\pi\frac{d\tau_m^\pi}{1-\tau_m^\pi}}{\Delta_1 + \Delta_2 - 1} \tag{50}$$

[illegible] (49) [illegible] (43) [illegible] (44), [illegible] (50) [illegible] (45) [illegible] (46) [illegible]

[illegible]:

$$\frac{dz_H^w}{z_H^w}\frac{1}{"_H^w} = -\frac{(1- \quad) \quad (\Delta_2 - 1)E_{(k)}\Big("_k^w\frac{d\tau_k^w}{1-\tau_k^w} \quad + \Delta_1 E_{(m)}\Big("_m^\pi\frac{d\tau_m^\pi}{1-\tau_m^\pi}}{\Delta_1(\Delta_1 + \Delta_2 - 1)}$$

$$+\frac{(\Delta_1 + \Delta_2 - 1) \quad \frac{d\tau_L^w}{1-\tau_L^w}(1- \quad)\frac{\delta_L l_L}{{}_k \delta l}"_L^w - \frac{d\tau_H^w}{1-\tau_H^w}\Big(1 + (1- \quad)\frac{\delta_L l_L}{{}_k \delta l}"_L^w}{\Delta_1(\Delta_1 + \Delta_2 - 1)}; \tag{51}$$

$$\frac{dz_L^w}{z_L^w}\frac{1}{"_L^w} = -\frac{(1- \quad) \quad (\Delta_2 - 1)E_{(k)}\Big("_k^w\frac{d\tau_k^w}{1-\tau_k^w} \quad + \Delta_1 E_{(m)}\Big("_m^\pi\frac{d\tau_m^\pi}{1-\tau_m^\pi}}{\Delta_1(\Delta_1 + \Delta_2 - 1)}$$

$$+\frac{(\Delta_1 + \Delta_2 - 1) \quad -\frac{d\tau_L^w}{1-\tau_L^w}\Big(1 + (1- \quad)\frac{\delta_H l_H}{{}_k \delta l}"_H^w \quad + \frac{d\tau_H^w}{1-\tau_H^w}(1- \quad)\frac{\delta_H l_H}{{}_k \delta l}"_H^w}{\Delta_1(\Delta_1 + \Delta_2 - 1)}; \tag{52}$$

$$\frac{dz_H^\pi}{z_H^\pi}\frac{1}{''^\pi_H} = -\frac{\Delta_2 E_{(k)}\Big(''^w_k \frac{d\tau_k^w}{1-\tau_k^w} \quad + (\Delta_1 - 1)E_{(m)}\Big(''^\pi_m \frac{d\tau_m^\pi}{1-\tau_m^\pi}}{\Delta_2(\Delta_1+\Delta_2-1)}$$

$$+\frac{(\Delta_1+\Delta_2-1) \quad \frac{d\tau_L^\pi}{1-\tau_L^\pi} \quad \frac{v_L q_L}{_k\, vq} ''^\pi_L - \frac{d\tau_H^\pi}{1-\tau_H^\pi}\Big(1+ \quad \frac{v_L q_L}{_k\, vq} ''^\pi_L}{\Delta_2(\Delta_1+\Delta_2-1)}; \qquad (53)$$

$$\frac{dz_L^\pi}{z_L^\pi}\frac{1}{''^\pi_L} = -\frac{\Delta_2 E_{(k)}\Big(''^w_k \frac{d\tau_k^w}{1-\tau_k^w} \quad + (\Delta_1 - 1)E_{(m)}\Big(''^\pi_m \frac{d\tau_m^\pi}{1-\tau_m^\pi}}{\Delta_2(\Delta_1+\Delta_2-1)}$$

$$+\frac{(\Delta_1+\Delta_2-1) \quad -\frac{d\tau_L^\pi}{1-\tau_L^\pi}\Big(1+ \quad \frac{v_H q_H}{_k\, vq} ''^\pi_H \quad + \frac{d\tau_H^\pi}{1-\tau_H^\pi} \quad \frac{v_H q_H}{_k\, vq} ''^\pi_H}{\Delta_2(\Delta_1+\Delta_2-1)}; \qquad (54)$$

F [illegible] (51)-(54) [illegible] fi [illegible].

$$\begin{aligned}
\frac{dz_H^w}{d \;\, ^w_H}\frac{1}{z_H^w} &= \frac{''^w_H}{1- \;\, ^w_H} \quad \frac{(1- \quad)\frac{\delta_H l_H}{_k\,\delta l}''^w_H - (\Delta_1+\Delta_2-1)}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^w}{d \;\, ^w_H}\frac{1}{z_L^w} &= \frac{\frac{\varepsilon_H^w}{1-\tau_H^w}\frac{\delta_H l_H}{_k\,\delta l}''^w_L(1- \quad)}{\Delta_1+\Delta_2-1}\\
\frac{dz_H^\pi}{d \;\, ^w_H}\frac{1}{z_H^\pi} &= -\frac{\frac{\varepsilon_H^w}{1-\tau_H^w}\frac{\delta_H l_H}{_k\,\delta l}''^\pi_H}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^\pi}{d \;\, ^w_H}\frac{1}{z_L^\pi} &= -\frac{\frac{\varepsilon_H^w}{1-\tau_H^w}\frac{\delta_H l_H}{_k\,\delta l}''^\pi_L}{\Delta_1+\Delta_2-1}
\end{aligned} \qquad (55)$$

$$\begin{aligned}
\frac{dz_H^w}{d \;\, ^w_L}\frac{1}{z_H^w} &= \frac{\frac{\varepsilon_L^w}{1-\tau_L^w}\frac{\delta_L l_L}{_k\,\delta l}''^w_H(1- \quad)}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^w}{d \;\, ^w_L}\frac{1}{z_L^w} &= \frac{''^w_L}{1- \;\, ^w_L} \quad \frac{(1- \quad)\frac{\delta_L l_L}{_k\,\delta l}''^w_L - (\Delta_1+\Delta_2-1)}{\Delta_1+\Delta_2-1}\\
\frac{dz_H^\pi}{d \;\, ^w_L}\frac{1}{z_H^\pi} &= -\frac{\frac{\varepsilon_L^w}{1-\tau_L^w}\frac{\delta_L l_L}{_k\,\delta l}''^\pi_H}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^\pi}{d \;\, ^w_L}\frac{1}{z_L^\pi} &= -\frac{\frac{\varepsilon_L^w}{1-\tau_L^w}\frac{\delta_L l_L}{_k\,\delta l}''^\pi_L}{\Delta_1+\Delta_2-1}
\end{aligned} \qquad (56)$$

$$
\begin{aligned}
\frac{dz_H^w}{d\ _H^\pi}\frac{1}{z_H^w} &= -\frac{\frac{\varepsilon_H^\pi}{1-\tau_H^\pi}\frac{v_H q_H}{\ _m vq}\ ''^w_H(1- \ \ )}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^w}{d\ _H^\pi}\frac{1}{z_L^w} &= -\frac{\frac{\varepsilon_H^\pi}{1-\tau_H^\pi}\frac{v_H q_H}{\ _m vq}\ ''^w_L(1- \ \ )}{\Delta_1+\Delta_2-1}\\
\frac{dz_H^\pi}{d\ _H^\pi}\frac{1}{z_H^\pi} &= \frac{''^\pi_H}{1-\ \ _H^\pi}\ \frac{''^\pi_H\frac{v_H q_H}{\ _m vq}-(\Delta_1+\Delta_2-1)}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^\pi}{d\ _H^\pi}\frac{1}{z_L^\pi} &= \frac{\frac{\varepsilon_H^\pi}{1-\tau_H^\pi}\frac{v_H q_H}{\ _m vq}\ ''^\pi_L}{\Delta_1+\Delta_2-1}
\end{aligned}
\tag{57}
$$

$$
\begin{aligned}
\frac{dz_H^w}{d\ _L^\pi}\frac{1}{z_H^w} &= -\frac{\frac{\varepsilon_L^\pi}{1-\tau_L^\pi}\frac{v_L q_L}{\ _m vq}\ ''^w_H(1- \ \ )}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^w}{d\ _L^\pi}\frac{1}{z_L^w} &= -\frac{\frac{\varepsilon_L^\pi}{1-\tau_L^\pi}\frac{v_L q_L}{\ _m vq}\ ''^w_L(1- \ \ )}{\Delta_1+\Delta_2-1}\\
\frac{dz_H^\pi}{d\ _L^\pi}\frac{1}{z_H^\pi} &= \frac{\frac{\varepsilon_L^\pi}{1-\tau_L^\pi}\frac{v_L q_L}{\ _m vq}\ ''^\pi_H}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^\pi}{d\ _L^\pi}\frac{1}{z_L^\pi} &= \frac{''^\pi_L}{1-\ \ _L^\pi}\ \frac{''^\pi_L\frac{v_L q_L}{\ _m vq}-(\Delta_1+\Delta_2-1)}{\Delta_1+\Delta_2-1}
\end{aligned}
\tag{58}
$$

[illegible]

[illegible]

$$
\begin{aligned}
\ _{\tau_k^w,\tau_m^\pi} W =\ & \ _k\ l_k \ -c_w\ \frac{z_k^w}{M(\ )\,w_k} \ + \ _m\ q_m\ -c_\pi\ \frac{z_m^\pi}{\frac{M(\theta)}{\theta}\ _m}\\
&+\ _H l_H c_w'\ \frac{z_H^w}{M(\ )\,w_H}\ +\ _L l_L c_w'\ \frac{z_L^w}{M(\ )\,w_L}\ + v_H q_H c_\pi'\ \frac{z_H^\pi}{\frac{M(\theta)}{\theta}\ _H}\ + v_L q_L c_\pi'\ \frac{z_L^\pi}{\frac{M(\theta)}{\theta}\ _L}\ + R\\
&+\ (\ \ _k\ l)M(\ )\ \frac{\ _H l_H}{\ _k\ l}\ ^w_H w_H + \frac{\ _L l_L}{\ _k\ l}\ ^w_L w_L + \frac{v_H q_H}{\ _m vq}\ ^\pi_H\ _H + \frac{v_L q_L}{\ _m vq}\ ^\pi_L\ _L\ ;
\end{aligned}
$$

[illegible]

$$
a = \frac{\ _H l_H}{\ _k\ l}\ ^w_H w_H + \frac{\ _L l_L}{\ _k\ l}\ ^w_L w_L \qquad b = \frac{v_H q_H}{\ _m vq}\ ^\pi_H\ _H + \frac{v_L q_L}{\ _m vq}\ ^\pi_L\ _L:
$$

$$\frac{@L}{@\ {}^{w}_{H}} =$$

$$= \quad {}_{k}\ l_k \ -\frac{c'_w}{M(\ )\,w_k}\frac{dz^w_k}{d\ {}^{w}_{H}} \ + \ {}_{m}\ q_m \ -\frac{c'_\pi}{\frac{M(\theta)}{\theta}\ {}_{m}}\frac{dz^\pi_m}{d\ {}^{w}_{H}}$$

$$+\frac{dz^w_H}{d\ {}^{w}_{H}}\frac{1}{M(\ )\,w_H}l_H c'_w \ \frac{z^w_H}{M(\ )\,w_H} \ + \ {}_{H}l_H c''_w \ \frac{z^w_H}{M(\ )\,w_H} \ \frac{1}{M(\ )\,w_H}\frac{dz^w_H}{d\ {}^{w}_{H}}$$

$$+\frac{dz^w_L}{d\ {}^{w}_{H}}\frac{1}{M(\ )\,w_L}l_L c'_w \ \frac{z^w_L}{M(\ )\,w_L} \ + \ {}_{L}l_L c''_w \ \frac{z^w_L}{M(\ )\,w_L} \ \frac{1}{M(\ )\,w_L}\frac{dz^w_L}{d\ {}^{w}_{H}}$$

$$+\frac{dz^\pi_H}{d\ {}^{\pi}_{H}}\frac{1}{\frac{M(\theta)}{\theta}\ {}_{H}}q_H c'_\pi \ \frac{z^\pi_H}{\frac{M(\theta)}{\theta}\ {}_{H}} \ + v_H q_H c''_\pi \ \frac{z^\pi_H}{\frac{M(\theta)}{\theta}\ {}_{H}} \ \frac{1}{\frac{M(\theta)}{\theta}\ {}_{H}}\frac{dz^\pi_H}{d\ {}^{\pi}_{H}}$$

$$+\frac{dz^\pi_L}{d\ {}^{\pi}_{L}}\frac{1}{\frac{M(\theta)}{\theta}\ {}_{L}}q_L c'_\pi \ \frac{z^\pi_L}{\frac{M(\theta)}{\theta}\ {}_{L}} \ + v_L q_L c''_\pi \ \frac{z^\pi_L}{\frac{M(\theta)}{\theta}\ {}_{L}} \ \frac{1}{\frac{M(\theta)}{\theta}\ {}_{L}}\frac{dz^\pi_L}{d\ {}^{\pi}_{L}}$$

$$+\left[\ {}_{k}\ \frac{l_k}{M(\ )\,w_k}\frac{dz^w_k}{d\ {}^{w}_{H}} \quad M(\ )+(\ {}_{k}\ l)M(\ ) \ \left(\frac{{}_{m}\ \frac{q_m}{\frac{M(\theta)}{\theta}\pi_m}\frac{dz^\pi_m}{d\tau^w_H}}{{}_{k}\ l} - \frac{(\ {}_{m}\ vq)\ {}_{k}\left(\frac{l_k}{M(\theta)\,w_k}\frac{dz^w_k}{d\tau^w_H}\right)}{(\ {}_{k}\ l)^2}\right)\right] (a+b)$$

$$+\ (\ {}_{k}\ l)M(\ )$$

$$_H l_H\, c_w'' \;\frac{z_H^w}{M(\ )\, w_H} \;\; \frac{1}{M(\ )\, w_H}\frac{dz_H^w}{d\ \ _H^w} + \;_L l_L\, c_w'' \;\frac{z_L^w}{M(\ )\, w}$$ [redacted]

$$+v_H q_H\, c_\pi'' \;\frac{z_H^\pi}{\frac{M(\theta)}{\theta}\ \ _H} \;\; \frac{1}{\frac{M(\theta)}{\theta}\ \ _H}\frac{dz_H^\pi}{d\ \ _H^w} + v_L q_L\, c_\pi'' \;\frac{z_L^\pi}{\frac{M(\theta)}{\theta}\ \ _L}$$ [redacted]

$$+\;(\ \ _k\ \ l)M(\ )\left[\frac{k\left(\frac{l_k}{M(\theta)\, w_k}\frac{dz_k^w}{d\tau_H^w}\right)}{_k\ \ l} + \frac{M'(\ )}{M(\ )}\;\frac{m}{\ }\right.$$ [redacted] $$\left.\frac{\frac{l_k}{M(\theta)\, w_k}\frac{dz_k^w}{d\tau_H^w}}{_k\ \ l}\right]\Big|\;(a+b) =$$

$$= \;_H l_H\, c_w'' \;\frac{z_H^w}{M(\ )\, w_H} \;\; \frac{1}{M(\ )\, w_H}\frac{dz_H^w}{d\ \ _H^w}\frac{z_H^w}{z_H^w}\frac{M(\ )\,(}{c_w'(z_H^w=}$$ [redacted]

$$+ \;_L l_L\, c_w'' \;\frac{z_L^w}{M(\ )\, w_L} \;\; \frac{1}{M(\ )\, w_L}\frac{dz_L^w}{d\ \ _H^w}\frac{z_L^w}{z_L^w}\frac{M(\ )\,(1-\ \ _L^w)w_L}{c_w'(z_L^w = M(\ )\, w_L)}$$

$$+v_H q_H\, c_\pi'' \;\frac{z_H^\pi}{\frac{M(\theta)}{\theta}\ \ _H} \;\; \frac{1}{\frac{M(\theta)}{\theta}\ \ _H}\frac{dz_H^\pi}{d\ \ _H^w}\frac{z_H^\pi}{z_H^\pi}\frac{M(\ )}{\ }\frac{(1-\ \ _H^\pi)\ \ _H}{c_\pi'(z_H^\pi = \ \ _H)}$$

$$+v_L q_L\, c_\pi'' \;\frac{z_L^\pi}{\frac{M(\theta)}{\theta}\ \ _L} \;\; \frac{1}{\frac{M(\theta)}{\theta}\ \ _L}\frac{dz_L^\pi}{d\ \ _H^w}\frac{z_L^\pi}{z_L^\pi}\frac{M(\ )}{\ }\frac{(1-\ \ _L^\pi)\ \ _L}{c_\pi'\left(z_L^\pi = \frac{M(\theta)}{\theta}\ \ _L\right.}$$

$$+\;(\ \ _k\ \ l)M(\ )\left[\quad k\left(\frac{l_k}{M(\theta)\, u\text{wH}}\right.\right.$$

$$= \quad {}_{H} l_{H} \frac{1}{''^{w}_{H}} \frac{dz^{w}_{H}}{d \; {}^{w}_{H}} \frac{1}{z^{w}_{H}}$$

$_{H}^{\pi}$, $_{L}^{\pi}$

$$(\Delta_1 + \Delta_2 - 1) \quad (1 - \quad)^{1 - \quad {}_{L}^{w}}$$

F

$$= \left(1 - \frac{\left[\begin{array}{l} \left(\frac{\delta_H l_H}{k\,\delta l}\frac{v_L q_L}{m\,vq} + \frac{\delta_L l_L}{k\,\delta l}\frac{v_H q_H}{m\,vq}\right) w_{HH} + \left(\frac{\delta_H l_H}{k\,\delta l}\frac{v_H q_H}{m\,vq} + \frac{\delta_L l_L}{k\,\delta l}\frac{v_L q_L}{m\,vq}\right) w_{HL} \quad {}^{w}_{H} \\ + \left(\frac{\delta_H l_H}{k\,\delta l}\frac{v_H q_H}{m\,vq} + \frac{\delta_L l_L}{k\,\delta l}\frac{v_L q_L}{m\,vq}\right) w_{LH} + \left(\frac{\delta_H l_H}{k\,\delta l}\frac{v_L q_L}{m\,vq} + \frac{\delta_L l_L}{k\,\delta l}\frac{v_H q_H}{m\,vq}\right) w_{LL} \quad {}^{w}_{L} \\ + \left(\frac{\delta_H l_H}{k\,\delta l}\frac{v_L q_L}{m\,vq} + \frac{\delta_L l_L}{k\,\delta l}\frac{v_H q_H}{m\,vq}\right) {}_{HH} + \left(\frac{\delta_H l_H}{k\,\delta l}\frac{v_H q_H}{m\,vq} + \frac{\delta_L l_L}{k\,\delta l}\frac{v_L q_L}{m\,vq}\right) {}_{HL} \quad {}^{\pi}_{H} \\ + \left(\frac{\delta_H l_H}{k\,\delta l}\frac{v_H q_H}{m\,vq} + \frac{\delta_L l_L}{k\,\delta l}\frac{v_L q_L}{m\,vq}\right) {}_{LH} + \left(\frac{\delta_H l_H}{k\,\delta l}\frac{v_L q_L}{m\,vq} + \frac{\delta_L l_L}{k\,\delta l}\frac{v_H q_H}{m\,vq}\right) {}_{LL} \quad {}^{\pi}_{L} \end{array}\right]}{\frac{1-\tau_H^w}{\varepsilon_H^w} w_H + \frac{1-\tau_L^w}{\varepsilon_L^w} w_L + \frac{1-\tau_H^\pi}{\varepsilon_H^\pi} {}_{H} + \frac{1-\tau_L^\pi}{\varepsilon_L^\pi} {}_{L}}\right)^{-1};$$

$$= \left(1 - \frac{E_{(s)} w_{Hs} {}^{w}_{H} + E_{(s)} w_{Ls} {}^{w}_{L} + E_{(s)} {}_{sH} {}^{\pi}_{H} + E_{(s)} {}_{sL} {}^{\pi}_{L}}{\frac{1-\tau_H^w}{\varepsilon_H^w} w_H + \frac{1-\tau_L^w}{\varepsilon_L^w} w_L + \frac{1-\tau_H^\pi}{\varepsilon_H^\pi} {}_{H} + \frac{1-\tau_L^\pi}{\varepsilon_L^\pi} {}_{L}}\right)^{-1}; \qquad (64)$$

s $\left(\frac{\delta_H l_H}{k\,\delta l}\frac{v_L q_L}{m\,vq} + \ldots\right.$

c'

$$'' = \frac{1}{} \frac{c''}{c'} = \frac{1}{-1}.$$

, A, :

.

2. $c = A(\ ^{\gamma} + \ ^{\beta})$ ≥ 3, $<$, $c' = A(\ ^{\gamma-1} + \ ^{\beta-1})$,

$$c'' = A(\ (\ -1)\ ^{\gamma-2} + \ (\ -1)\ ^{\beta-2}) > 0;$$

$$'' = \frac{\ ^{\gamma-2} + \ ^{\beta-2}}{(\ -1)\ ^{\gamma-2} + (\ -1)\ ^{\beta-2}}.$$

A , , .

$$\frac{\partial ''}{\partial} = -\ ^{\beta+\gamma-5}(\ -\ )^2 < 0.$$

N ≥ 3 $= \ -1$. . F ,

< 1 $= 3$ $= 2$, > 1 $= 2$ $= 1$.

$\frac{1-\tau_H^w}{\varepsilon_H^w} w_H < \frac{1-\tau_L^w}{\varepsilon_L^w} w_L$, (66) -

. H $(1- \ _H^w) w_H < (1- \ _L^w) w_L$ $\ _H^w > \ _L^w$

(66) $\ _H^w w_H > \ _L^w w_L$. N $(E_{(m)}\ _{Hm}\ _m^{\pi} - E_{(m)}\ _{Lm}\ _m^{\pi})$. -

$\ _{HH} > \ _{LH}$ $\ _{HL} > \ _{LL}$ $(E_{(m)}\ _{Hm}\ _m^{\pi} - E_{(m)}\ _{Lm}\ _m^{\pi} > 0)$.

H (66) .

$(1- \ _H^w) w_H > (1- \ _L^w) w_L$, $\ _H > \ _L$, $''^w_H \leq ''^w_L$. □

Proof of Proposition 11.

fi (65).

fi

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fi (60)-(63)

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, . D

(60) (61)

$$(\Delta_1 + \Delta_2 - 1) \quad (1 - \quad) \frac{1 - \quad ^{w}}{''_{w}} w + \quad (w \quad ^{w} + \quad ^{\pi} - (1 - \quad) \bar{R}) \quad =$$

$$= (1 - \quad) \left[(1 - \quad ^{w}) w + \quad (w \quad ^{w\,''w} + \quad ^{\pi\,''w} - (1 - \quad) \bar{R}^{\,''w}) \right]$$

$-$

(68), (69), P 11.

P 12

(1 −), (1 −) = ↑,

, π = w ↓.□