DISCUSSION PAPERS IN ECONOMICS

Working Paper No. 03-16

Vertical Foreign Direct Investment, Knowledge Spillovers and the Global Growth: A Q-theory Approach

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November 2003

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Vertical Foreign Direct Investment, Knowledge Spillovers and the Global Growth: A Q-theory Approach

1. Introduction

2. Literature review

3. The Static Model

3.1 Consumers

$$U \quad = \quad \left(C_X^{\phi} C_Y^{-\phi}\right)$$

$$C_X \equiv \left\{ \int^{n+m+s} [c_i]^{-\quad\varepsilon} di \right\}^{\overline{-\quad\varepsilon}}$$

ε

ε

$$p = w \quad \alpha$$

$$< \alpha = \quad - \quad \varepsilon <$$

3.3 FDI

$$S^M = K^M \quad K^N + K^M$$

$$S^N = K^N \quad K^N + K^M \quad = \quad - S^M$$

$$S^S = K^S \quad K^N + K^M$$

$$S^M \quad - S^M \left[\frac{\pi^N}{F} - \frac{\pi^M}{F \quad + \Gamma} \right] =$$

$$\Rightarrow \quad \leq \Gamma \leq w^{\varepsilon -} \; -$$

$$C_X \equiv \left\{ \int^{n+m+s} [c_i]^{-\ \ \varepsilon} di \right\}^{\overline{-\ \ \varepsilon}} \qquad \varepsilon > \qquad \rho$$

$$E \ \ E = r - \rho$$

4.2 Knowledge Capital

a_I a_I

$$a_I = \frac{}{K^N + K^M \ \ + \lambda K^S + \mu n}$$

$$\geq \lambda \geq \qquad \mu \geq$$

$$K = K^{N} + K^{M} + K^{S}$$

λ

S^S

4.3 FDI

$$S^M \quad - S^M \left[\frac{\Pi^N}{F} - \frac{\Pi^M}{F \quad + \Gamma} \right] =$$

$$\Pi^i \equiv \int_{s=}^{\infty} e^{-rs} \pi_s^i ds \qquad\qquad i = N \;\; M \qquad \pi^i$$

$$\pi^M = \pi^S = \frac{-\alpha \;\; \phi \cdot E}{w^{-\varepsilon} n + m + s} = \frac{-\alpha \;\; \phi \cdot E}{K^N + K^M \quad w^{-\varepsilon} \quad - S^M \;\; + S^M + S^S}$$

$$\pi^N = \frac{-\alpha \;\; \cdot w^{-\varepsilon} \cdot \phi \cdot E}{w^{-\varepsilon} n + m + s} = \frac{-\alpha \;\; \cdot w^{-\varepsilon} \cdot \phi \cdot E}{K^N + K^M \quad w^{-\varepsilon} \quad - S^M \;\; + S^M + S^S}$$

$K^N + K^M$

$K^N + K^M$ $g^N = L_I A$ π^i

$$E = \frac{L^S + wL^N - wL_I}{-\phi \quad -\alpha}$$

L_I

$L_I =$

$E =$ $r = \rho$

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$$q \equiv V \quad F =$$

$$q^N \equiv V^N \quad F = q^M \equiv V^M \qquad + \Gamma \;\; F =$$

$$V_t \equiv \int_{s=t}^{\infty} e^{-r \;\; s-t} \; \pi_s^i ds = \pi \quad \rho + g^N \qquad i = N \;\; M$$

$$L_I \quad = \frac{L^S + wL^N \qquad -\alpha \;\; \phi A - w[\;\; -\phi \quad -\alpha \;\;]\rho\left[S^M + \quad + \Gamma \;\; S^S + \quad + \Gamma \;\; w^{-\varepsilon} S^N\right]}{w[\;\; -\phi \quad -\alpha \;\;]A\left[S^M + \quad + \Gamma \;\; S^S + \quad + \Gamma \;\; w^{-\varepsilon} S^N\right] + w\phi A \quad -\alpha}$$

$$g^N \quad = \frac{L^S + wL^N \qquad -\alpha \;\; \phi\left[\;\; + \mu \quad - S^M \;\; \right] - w[\;\; -\phi \quad -\alpha \;\;]\rho}{w}$$

$$\frac{\partial g}{\partial S^{M}} = \frac{L^{S} + wL^{N} \quad -\alpha \quad -\phi\mu}{w} \leq$$

$$g = K \quad K = g^{N} \qquad + S^{S}$$

$$g^{S} = \frac{K^{S}}{K^{S}} = \frac{\frac{j}{+\Gamma} K^{M}}{K^{S} + \frac{j}{+\Gamma} K^{M}}$$

$$= g^{N} = g = L_{I} A$$

5.2 Production of X

$$w \quad \alpha \qquad\qquad \alpha$$

$$P_{X} = \left[\left(\frac{}{\alpha} \right)^{-\varepsilon} s \quad + \left(\frac{}{\alpha} \right)^{-\varepsilon} m - j \cdot m \quad + \quad {}^{-\varepsilon} \cdot j \cdot m + \left(\frac{w}{\alpha} \right)^{-\varepsilon} n \right]^{\overline{-\varepsilon}}$$

g^N g^N

$\Pi^N = \pi^N \quad \rho + g^N$

$\Pi^M = \pi^M \quad \rho + j + g^N$

—

Result 1: A lower imitation rate leads to a higher rate of multinationalization and a lower level of investment in innovation. As a result, knowledge capital grows more slowly in the long run.

5.3.3 Disadvantage cost, MNCs share and long-run growth rate

Γ

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Result 3: Increases in the wage gap or elasticity of substitution between varieties increases the rate of multinationalization, decreases the investment level, and decreases the long-run growth rate.

5.4 Solving for long-run GDP growth rate

$E =$

$L_I =$

$real\ \ GDP$

$$g_{GDP} = -\frac{P}{P} = -\left[\quad -\phi \quad \cdot \frac{P_Y}{P_Y} + \phi \cdot \frac{P_X}{P_X} \right] = -\phi \cdot \frac{P_X}{P_X}$$

$$\frac{P_X}{P_X} == \frac{}{-\varepsilon} g$$

$$g_{GDP} = \frac{\phi}{\varepsilon -} g$$

5. Conclusions

Appendix1 The Static Model

Consumer's problem:

$$U = \left(C_X^{\phi} C_Y^{-\phi}\right) \qquad C_X \equiv \left\{\int^{n+m+s} [c_i]^{-\ \varepsilon} di\right\}^{\overline{-\ \varepsilon}}$$

$$P_X C_X + C_Y = E$$

$$U = C_X^{\phi} C_Y^{-\phi} + \lambda[E - P_X C_X + C_Y]$$

$$1 \quad \frac{\partial U}{\partial C_X} = \quad \Rightarrow \quad \frac{\phi C_X^{\phi -} C_Y^{-\phi}}{C_X^{\phi} C_Y^{-\phi}} = -\lambda P_X$$

$$2 \quad \frac{\partial U}{\partial C_Y} = \quad \Rightarrow \quad \frac{-\phi\ C_X^{\phi} C_Y^{-\phi}}{C_X^{\phi} C_Y^{-\phi}} = -\lambda$$

3 —

$$5\quad \frac{\partial U}{\partial c_b} = \quad \Rightarrow \quad \frac{\phi C_X^{\phi -} C_Y^{-\phi}}{C_X^{\phi} C_Y^{-\phi}} \cdot \frac{}{-\frac{}{\varepsilon}} \cdot \left(\int^{n+m+s} c_i^{-\frac{}{\varepsilon}} di \right)^{\frac{}{-\frac{}{\varepsilon}}} \cdot \quad -\frac{}{\varepsilon} = -\lambda P_b$$

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$$M \quad S \quad \frac{\quad}{\quad} \quad \frac{\qquad -\big)}{\left(\frac{w}{\quad}\right) n + \left(\frac{\quad}{\quad}\right) m + \left(\frac{\quad}{\quad}\right) s} \cdot \quad \cdot E = \frac{- \quad \cdot E}{w \quad n + m + s}$$

1.4 FDI or not?

$$\frac{\pi^M}{+\Gamma\ F} \geq \frac{\pi^N}{F}$$

$$\frac{\qquad\qquad E}{w \quad n \quad m \quad s} \qquad \frac{w \quad E}{w \quad n \quad m+s}$$

2 $\Leftrightarrow$

$$\Rightarrow C_Y^N \cdot \frac{}{w} = L_Y^N$$

3 $\Leftrightarrow$

$$\Rightarrow n \cdot c_i^N = n \cdot \frac{\left(\frac{w}{\alpha}\right)^{-\varepsilon} \phi \cdot E}{\left(\frac{w}{\alpha}\right)^{-\varepsilon} n + \left(\frac{}{\alpha}\right)^{-\varepsilon} m + \left(\frac{}{\alpha}\right)^{-\varepsilon} s}$$

4 $\Leftrightarrow$

$$\Rightarrow m \cdot c_i^M = n \cdot \frac{\left(\frac{}{\alpha}\right)^{-\varepsilon} \phi \cdot E}{\left(\frac{w}{\alpha}\right)^{-\varepsilon} n + \left(\frac{}{\alpha}\right)^{-\varepsilon} m + \left(\frac{}{\alpha}\right)^{-\varepsilon} s}$$

5 $\Leftrightarrow$

$$\Rightarrow s \cdot c_i^S = n \cdot \frac{\left(\frac{}{\alpha}\right)^{-\varepsilon} \phi \cdot E}{\left(\frac{w}{\alpha}\right)^{-\varepsilon} n + \left(\frac{}{\alpha}\right)^{-\varepsilon} m + \left(\frac{}{\alpha}\right)^{-\varepsilon} s}$$

$$L = C_Y^S + C_Y^N \cdot \frac{}{w} + n \cdot \frac{\left(\frac{w}{\alpha}\right)^{-\varepsilon} \phi \cdot E}{\left(\frac{w}{\alpha}\right)^{-\varepsilon} n + \left(\frac{}{\alpha}\right)^{-\varepsilon} m + \left(\frac{}{\alpha}\right)^{-\varepsilon} s} + m \cdot \frac{\left(\frac{}{\alpha}\right)^{-\varepsilon} \phi \cdot E}{\left(\frac{w}{\alpha}\right)^{-\varepsilon} n + \left(\frac{}{\alpha}\right)^{-\varepsilon} m + \left(\frac{}{\alpha}\right)^{-\varepsilon} s}$$

$$+ s \cdot \frac{\left(\frac{}{\alpha}\right)^{-\varepsilon} \phi \cdot E}{\left(\frac{w}{\alpha}\right)^{-\varepsilon} n + \left(\frac{}{\alpha}\right)^{-\varepsilon} m + \left(\frac{}{\alpha}\right)^{-\varepsilon} s}$$

$$L = C_Y^S + C_Y^N \cdot \frac{}{w} + n \cdot \frac{\alpha w^{-\varepsilon} \phi \cdot E}{w^{-\varepsilon} n + m + s} + m \cdot \frac{\alpha \phi \cdot E}{w^{-\varepsilon} n + m + s} + s \cdot \frac{\alpha \phi \cdot E}{w^{-\varepsilon} n + m + s}$$

$$L^S = C_Y^S + m \cdot \frac{\alpha \phi \cdot E}{w^{-\varepsilon} n + m + s} + s \cdot \frac{\alpha \phi \cdot E}{w^{-\varepsilon} n + m + s}$$

Appendix2 The Benchmark Model

2.1 Growth rate of knowledge capital held by the North.

$$N \qquad \frac{\qquad}{N \qquad M} \qquad \frac{I\left[\quad N \qquad M \qquad S \qquad N\right]}{N \qquad M}$$

$$F = wa_I \qquad a_I = \frac{L_I}{g^N} \cdot \frac{}{K^N + K^M} = \frac{L_I}{L_I A \; K^N + K^M} = \frac{}{A \; K^N + K^M}$$

$$= \frac{+\Gamma \; w}{A \; K^N + K^M}$$

$$Numerator = \frac{-\alpha \; \phi \cdot E}{w^{-\varepsilon} n + m + s \quad \rho + g^N}$$

$$\frac{wL}{}$$

$$g^N \quad L_I A \qquad \frac{}{L \;\; A}$$

Appendix3   Math calculation and derivation for the imitation model

3.1 Growth rate of southern knowledge capital.

$$g^S = \frac{K^S}{K^S} = \frac{\frac{j}{+\Gamma} K^M}{K^S + \frac{j}{+\Gamma} K^M}$$

$$K^N + K^M$$

$$g^S = \frac{\frac{j}{+\Gamma} \cdot \frac{K^M}{K^N + K^M}}{S^S + \frac{j}{+\Gamma} \cdot \frac{K^M}{K^N + K^M}}$$

S^S

$$E = \frac{L^S + wL^N - wL_I \quad \cdot \left(w^{-\varepsilon} S^N + \frac{S^M}{+\Gamma} - j \cdot \frac{S^M}{+\Gamma} + S^S + \alpha^{-\varepsilon} j \cdot \frac{S^M}{+\Gamma} \right)}{w^{-\varepsilon} S^N + \frac{S^M}{+\Gamma} - j \cdot \frac{S^M}{+\Gamma} + S^S \quad \cdot [\; - \quad - \alpha \; \phi] + \alpha^{-\varepsilon} j \cdot \frac{S^M}{+\Gamma}}$$

3.3 Solve for equilibrium investment level in R&D with imitation:

$$q^M \equiv \frac{V^M}{+\Gamma \;\; F} = \frac{\frac{\pi^M}{\rho + j + g}}{+\Gamma \;\; F} =$$

$$= \frac{+\Gamma \;\; w}{A \;\; K^N + K^M}$$

$$Numerator = \frac{-\alpha \;\; \phi \cdot E}{w^{-\varepsilon} n + m - jm + \alpha^{-\varepsilon} \cdot jm + s \quad \cdot \;\; \rho + j + g}$$

$$g \quad L_I A$$

$$Numerator = \frac{-\alpha \;\; \phi \cdot \frac{L^S + wL^N - wL_I \quad \cdot \left(w^{-\varepsilon} n + m - jm + s \quad + \alpha^{-\varepsilon} jm \right)}{w^{-\varepsilon} n + m - jm + s \quad \cdot [\; - \quad - \alpha \;\; \phi] + \alpha^{-\varepsilon} jm}}{\rho + L_I A + j \quad w^{-\varepsilon} n + m - jm + s \quad + \alpha^{-\varepsilon} jm}$$

$$= \frac{L^S + wL^N - wL_I \qquad - \alpha \;\; \phi}{\rho + L_I A + j \;\; \left\{ w^{-\varepsilon} n + m - jm + s \quad \cdot [\; - \quad - \alpha \;\; \phi] + \alpha^{-\varepsilon} jm \right\}}$$

$$K^N + K^M$$

$$L^S + wL^N - wL_I \qquad - \alpha \;\; \phi A =$$
$$\rho + L_I A + j \;\; w \left\{ \left[w^{-\varepsilon} \quad + \Gamma \;\; S^N + S^M - jS^M + \quad + \Gamma \;\; S^S \right] \cdot [\; - \quad - \alpha \;\; \phi] + \alpha^{-\varepsilon} jS^M \right\}$$

$$L_I$$

$$L_I = \frac{L^S + wL^N \quad - \alpha\ \phi[\ + \mu \quad - S^M \ ] - \ \rho + j\ w\{[w^{-\varepsilon} \quad + \Gamma \quad - S^M \ + S^M - jS^M]\cdot[\ - \quad - \alpha\ \phi] + \alpha^{-\varepsilon} jS^M\}}{w[\ + \mu \quad - S^M \ ]\{[w^{-\varepsilon} \quad + \Gamma \quad - S^M \ + S^M - jS^M]\cdot[\ - \quad - \alpha\ \phi] + \alpha^{-\varepsilon} jS^M + \quad - \alpha\ \phi\}}$$

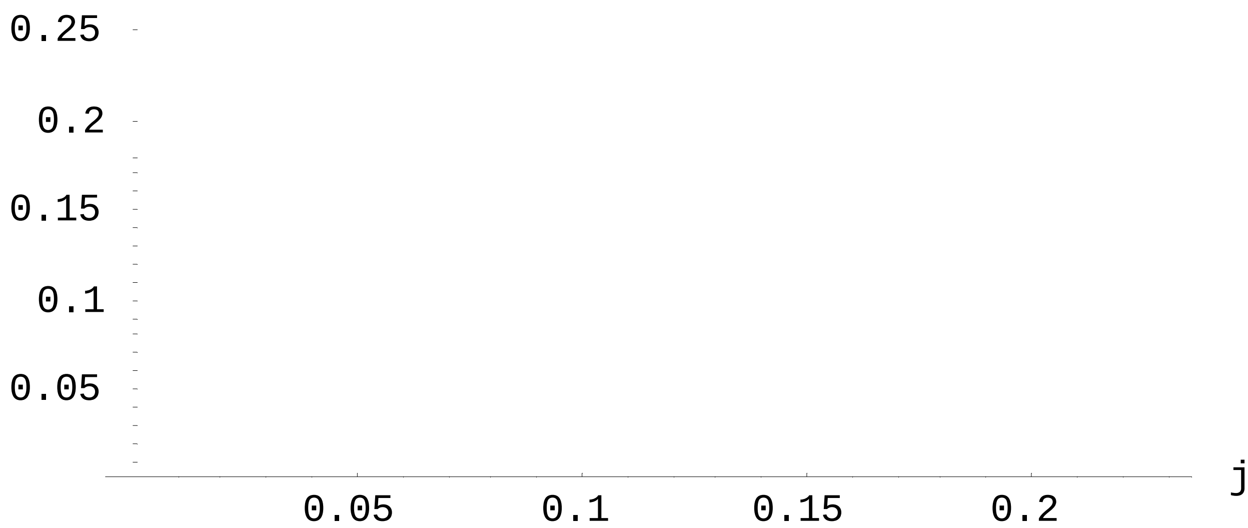
3.4 Solution of share of MNCs (from Mathematica)

$$= \ ^{\varepsilon} \cdot [\quad + \Gamma\ + \quad - \quad - \ ^{\varepsilon -} + \Gamma\ + \quad + \quad - \quad + \quad - \ ^{\varepsilon -} + \Gamma\ ]\cdot$$

$$- \quad ^{\varepsilon}\ + \quad ^{\varepsilon}\ + \Gamma - \ ^{\varepsilon -}\ + \ ^{\varepsilon -}\ - \quad - \ ^{\varepsilon -}\ + \ ^{\varepsilon -}\ - \quad - \ ^{\varepsilon} \ ^{+\varepsilon}\phi \ -$$

$$- \quad \alpha^{\varepsilon}\Gamma\phi \ - \alpha\ + \quad + \quad \alpha^{\varepsilon}\mu\phi \ - \alpha \quad + \Gamma - \ ^{\varepsilon -} \ ^{-}$$

$$g_{GDP} = -\left[\quad -\phi \quad \cdot \frac{P_Y}{P_Y} + \phi \cdot \frac{P_X}{P_X} \right] = -\phi \cdot \frac{P_X}{P_X} = \frac{\phi}{\varepsilon -} g$$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