Anna Rubinchik-Pessach and Roberto M. Samaniego[†]

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We investigate the relationship between polity formation and the level of economic activity. We posit a dynamic search environment in which opportunities for mutually beneficial trade may be hampered by theft. Agents search for potential trading partners and, if matched, optimally choose whether to attempt to trade or to steal from each other. The excludability of goods – in the form of respected property rights – is endogenously determined as a result.

We compare the equilibria of this game under anarchy to those of an identical environment in which there is a "government" in the minimal sense of an agency that protects property rights. In exchange for protection, agents pay a certain amount to enter the market. We find that agents' willingness to pay for these services – i.e., to be taxed – is increasing in pot p0o(t)-0.3(o)i12.2(sil367.24.4g)1(a)23.2(sii3

H11, H41, K42, O17

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Much has been written regarding the beneficial effects of government on the level of economic activity (see Shleifer (1998), Besley (1995) inter alia). There is a consensus that a necessary condition to modern growth is the existence of an incentive system that benefits those who incur the costs of productive endeavor. In particular, a precondition for the voluntary exchange of goods and services in a non-autarkic economy of non-altruistic agents is the existence of enforced property rights, such that agents may reap the benefits of costs that they incur. For example, if levels of theft are high, agents will not have an incentive to produce translatable goods, and will find it optimal to revert to autarky or subsistence.

The conventional rationale for government to arise in an autarkic anarchy is the presence of returns to scale in the protection technology. For instance, in Grossman (2002), a government centralizes protection activities with the aim of decreasing the probability that a good belonging to an individual is expropriated by another. Centralizing protection, therefore, can eliminate the over-investment of productive rethat enforces property rights can arise through market-like mechanisms. In ASU, agents are willing to pay for the services of a “protection agency” (or a “minimal government”) which adjudicates in disputes and protects property rights. We adopt a simple environment in which disputes are clear-cut in that they only arise in the case of overt theft, and derive demand for such an entity from economic primitives.[1] Unlike Nozick, we do not envision any notion of entitlements beyond institutions, and explicitly derive the conditions under which both property and, indeed, trade itself, may emerge.[2] We ignore the possibility that government may serve other functions in order to focus on its role protecting property rights.

The model addresses some fundamental issues regarding the nature of goods and activity,"[4]

for β $(0, 1)$. Thus, each agent seeks a proportion β of the goods produced by the others. Note that, if agent i likes the good produced by agent j, the converse also holds. Therefore, if two agents are anonymously matched pairwise, the conditional to exchange have no purpose in staying in Market town since, lacking a good, nobody will approach them close enough for any interaction. Theft, being their only option, has no opportunity to materialize.

We also assume that agents' histories are unobservable, and that the probability of meeting the same agent twice is zero. We also focus on stationary equilibria, in which agents do not adopt strategies that are contingent upon their own histories.[7] As market. If the partner has chosen to rob, she looses her good and leaves empty handed. Thus, the instantaneous expected payoff conditional on a match is $u^a(\text{trade}) \equiv \gamma G$. If, instead, the agent decides to rob, she deprives her partner of his good and consumes it for a payoff of G. In this case she remains in the marketplace, maintaining possession of her own produce for a continuation payoff V^a in the following period. Denote by V^a the value attached to a tradeable good under anarchy. Thus, the encounter yields $W^a \equiv G + \delta V^a$.[9] If, however, the partners simultaneously attempt to rob, each succeeds with probability one half. In this case, the payoff conditional on the match is $u^a(\text{rob}) \equiv \gamma W^a + (1-\gamma)\frac{1}{2}W^a$. Hence, in equilibrium,

$$V^a = \beta \max\{u^a(\text{trade}), u^a(\text{rob})\} + (1-\beta)\delta V^a. \tag{2}$$

It is straightforward to demonstrate that

The only equilibrium under anarchy is $\gamma = 0$.

The Market town becomes a "Den of Thieves": farmers bring their produce to market only to be robbed or to steal from others. They use their good, like a "bait," to attract partners with desired goods. Having verified the "double coincidence of

ω) and her good re-instated, thus, enabling her to stay in the marketplace yielding $\omega\delta V^g$. The expected value of an appropriate match for the robber is then $u^g(\text{rob}) = \gamma W^g(\omega, c) + (1 - \gamma)\left(\frac{1}{2}W^g(\omega, c) + \frac{1}{2}\omega\delta V^g\right)$.

The value traders attach to the good they bring to market is

$$V^g = \beta \max\{u^g(\text{trade}), u^g(\text{rob})\} + (1 - \beta)\delta V^g \tag{3}$$

Recall that an equilibrium is a fraction of fair traders γ consistent with the optimal behavior of all potential traders. In a stationary equilibrium, an agent will choose either one action (to rob or to trade) "forever", or she will be indifferent between the two. We are interested in understanding what equilibria can be "induced" by a protection agency using punishment c and intensity ω of observing trades.

We start with two simple observations.[14] First, if c is high enough, it is an equilibrium for everybody to trade fairly. Second, in spite of the minimal state, the "Den of Thieves" equilibrium exists, if detection and retribution are lenient. Indeed, both "corner" equilibria exist for some range of punishments c.

In addition, there are 'interior' equilibria in which fair traders and robbers are present in the Market Town. To find these, we look at the difference $F(\gamma; c)$ between the value of tradeable goods for perpetual fair traders V_t^g and that of chronic robbers V_r^g.[15] [16]

Figure 2 plots $F(\gamma; c)$ for three different values of c, keeping other parameters fixed.[17] Any roots of F in $(0, 1)$ correspond to interior equilibria.

Figure 2 suggests that there are values of c that potentially generate two interior equilibria. Higher values of c are associated with only one interior equilibrium, while $\gamma = 1$ is also an equilibrium since $F(1, c) > 0$. On the other hand, punishments may also be so low that $\gamma = 0$ is the only equilibrium. According to the Figure 2, this "no trade" equilibrium is also present in the two cases mentioned above, since $F(0, c) < 0$ in all the cases depicted.

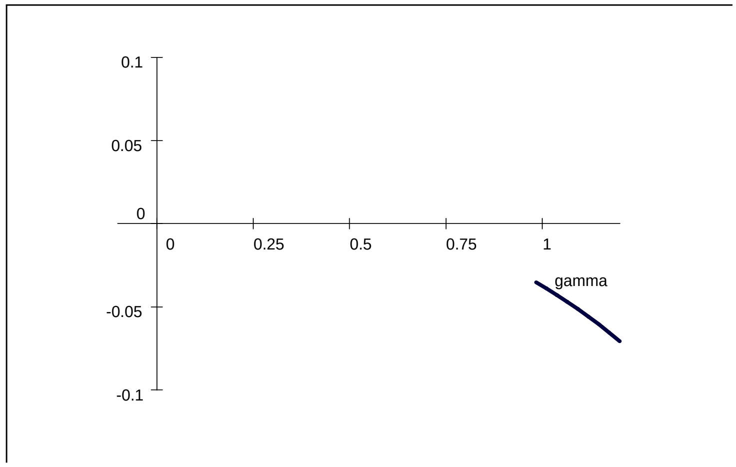
Finally, it is not surprising that, if c is high enough, $\gamma = 1$ is the unique equilib-

[14] See lemmata 14, 15 in Appendix A.2.

[15] See Appendix A.2 for the validity of this approach.

[16] More precisely, we use a function $F(\gamma; c)$ that has the same sign as this difference. See the detailed definitions in Appendix A.1.

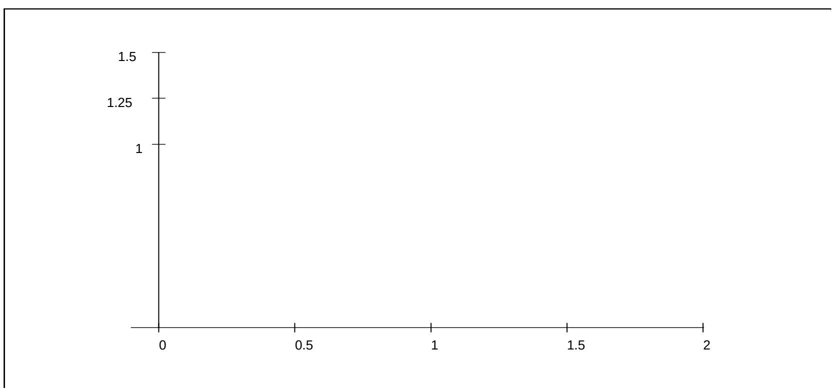
[17] Parameter values in this example are $\omega = 0.5$, $G = 1$, $\beta = 0.5$, and $\delta = 0.9$.

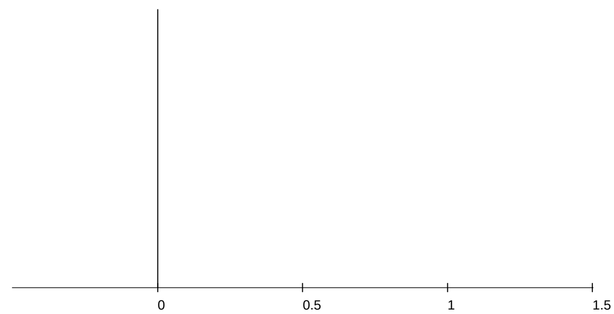

0.1
0.05
0
-0.05
-0.1
0
0.25
0.5
0.75
1
gamma

If $\omega \quad \frac{\beta\delta}{1 \quad \delta+\beta\delta}$, then $\gamma = 1$ (full trade) is an equilibrium even if $c = 0$.

By corollary 3, if the detection rate is high, there is no need to inflict direct cost c on the observed robbers to prevent robbery altogether. The fact that the government reinstates the stolen item to the owner obliges the thieves to wait for a future opportunity before stealing or trading, which is a su cient deterrent in itself. If agents are impatient or meetings are rare, this e ect is exacerbated.

The relation between the equilibrium values of γ and punishment c can be conveniently represented graphically. Figure 3 depicts an example of an environment described in the second part of Proposition 2. Observe that $\gamma_L(c)$ approaches zero as c approaches $\bar{c}(\omega)$, and that $\gamma_L(c)$ and $\gamma_H(c)$ converge as c decreases towards $\underline{\underline{c}}(\omega)$.

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In addition observe that more severe punishments are needed to discourage robbery in “patient” societies. The reason is related to Corollary 3. A high discount rate

Proposition (4) states that the inhabitants of Farmland are ready to pay for the access to the Market Town, if the induced proportion of fair traders is above a certain threshold, in other words, if the marketplace is safe enough. It is also easy to check that the demand grows with that enforces property rights. In some cases the Minimal State, viewed as a unit, may be motivated to increase the severity of punishment as the gains from trade increase, if it is capable of capturing part of the additional willingness to pay for protection that arises as a result (so long as they are not constrained by the mores of the cultural environment, or by technology).[20]

γ may be di erent from zero. We denote this u^{cg} (trade) = γG, where the superscript cg refers to "corrupt government".

As for robbers, the value of their good decreases as well. In case of an unsuccessful theft (which occurs with probability $\frac{1}{2}$ conditional on meeting another robber)+I62.4-8.6hey leave the marketplace empty handed. Thus,

$$u^{cg}(rob) = \gamma W^{g}(\omega, c) + (1 \quad \gamma) \frac{1}{2} W^{g}(\omega, c) \qquad (7)$$

where $W^{g}(\omega, c) = G + \omega(\delta V c + G) = (1 \quad \omega) G \quad \omega c + \delta V^{gc}$.

Assuming that government agents take bribes changes the equilibria of the model. However, their structure remains identical toe under aon-corrupt minimal state.

Assume protection agents can receive maximal bribe, c + G. If $\beta\delta$ + δ 1 , then the set of equilibria ^{cg}(c) can be described as follows:

$$^{cg}(\Gamma \quad c) = {}^{cg}_{L}(\{0 \quad c), \gamma\_(, 1 \quad \omega)\} \text{ if } , \bar{c}(c \; [\omega)] \quad c$$

{0 } _(if ω) c < c

{1 } if c > $\bar{c}$(

(If $\beta\delta$ + δ 1 > , then the set of equilibria c) can be described as follows:

{0 } (if ω) c < c

$$^{cg}(\Gamma \quad c) = {}^{cg}_{L}(\{0 \; {}^{cg}_{H}(c), \gamma \quad c), \gamma \; (\}(\text{if } \omega) \quad \omega c)) \; [c \quad c$$

$^{cg}_{L}$({0 c) , γ , 1(} ω) if , $\bar{c}$(c [ω)] c

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$^{cg}_{L}$($^{cg}_{H}$(c) c) , γ) $^{cg}_{L}$(0 $^{cg}_{H}$(0), γ with c < γ . γ

The lower bound c(ω) γ = 1 o equilibria does not vary, regardless of whether corruption introduces the possibility of a minimal state for punishments that were previously too low.

Assume $\beta\delta + \delta > 1$. Then $\underline{\underline{c}}^{cg}(\omega) < \underline{\underline{c}}(\omega)$. Moreover, if c $\underline{\underline{c}}(\omega), \underline{c}(\omega)$. Then, $\gamma_L^{cg}(c) < \gamma_L(c)$, and $\gamma_H^{cg}(c) > \gamma_H(c)$.

Suppose that c $\underline{\underline{c}}^{cg}(\omega), \underline{\underline{c}}(\omega)$. Trade can be supported in equilibrium under corruption, whereas it cannot in its absence.

That corrupt government leads to there being less theft in a stable equilibrium (i.e., that $\gamma_H^{cg}(c) > \gamma_H(c)$) may appear surprising. For a given γ, however, thieves are always better o when government is corrupt. Hence, for payo s to be equal across strategies, γ must be higher, to “encourage” the fair traders.[23]

In spite of corruption, the government continues to play a role that agents are willing to pay for. Let $D^{cg}(\gamma, G)$ be the willingness to pay for a government in the case of the maximal bribe $c + G$. This demand for government is defined as the di erence between the value of a tradeable good in the protected marketplace and that under anarchy, as in the previous section. This demand is still increasing in the gains from trade, provided the safety of the marketplace (γ) is kept constant.

Assume $\beta\delta + \delta > 1$; c $\underline{\underline{c}}(\omega), \underline{c}(\omega)$ and $\gamma_H^{cg}(G, c) > \underline{\gamma}^{cg}$, where $1 > \underline{\gamma}^{cg} > \underline{\gamma} > 0$. Then, An interesting question is whether protection agents will engage in corruption, if they are availed of the choice. We modify the model to address this question by simply assuming that the agency is a revenue-maximizing agent.[24] The agent is able to either charge a lump-sum tax τ, or to appropriate a fraction D of demand D in return for its services. We focus on stable equilibria with trade.

First, if $c \quad [\underline{c}(\omega), \overline{c}(\omega)]$, the only stable trading equilibrium is $\gamma = 1$, both in corrupt and in non-corrupt environments. Therefore payo s, demand for government and government revenue are identical, since no bribes will be paid. A more interesting case is where $\beta\delta + \delta > 1$, $c \quad \underline{\underline{c}}(\omega), \underline{c}(\omega)$ so that, in both environments, there are

In this section, we close the model by extending it to an environment with repeated production. Although the value function does not have as clear an interpretation as before[25]; however, this environment is better suited to addressing the welfare implications of introducing the minimal state. In the extended "economy" we can determine the quantity produced and consumed in a steady state and thus, compare this quantity across equilibria with and without the government. As will be shown below, the Minimal State, by protecting property rights, induces more production and consumption (per a time period), thus increasing well-being of the farmers.

Additional goods produced with enforced property rights can be thought of as a "real" source of the willingness to pay for the government.

We now turn to a more detailed analysis. Instead of a one[redacted]
farmers may travel between locations at their discretion. When in Farmland, they may choose to produce the translatable good and, if they leave Market town, they are free to return to Farmland to obtain more of the translatable good. Hence, in any trading equilibrium, value functions represent the expected value not only of holding one good but to being a farmer for the indefinite future, namely, producing a good in a Farmland and selling it in the Market town. Travel between locations takes one period, and is otherwise costless. If they choose to remain in Farmland, they earn utility ψ – which is set to zero for now.

Normalize the total mass of agents to unity. The fraction of people in Market town at time t is denoted by n_t. Let x^t_{in} be the fraction of people entering Market town, and x^t_{out} be the fraction of people leaving the market place at time t. The evolution of the population in Market town n_t is then

$$n_{t+1}$$

agents.

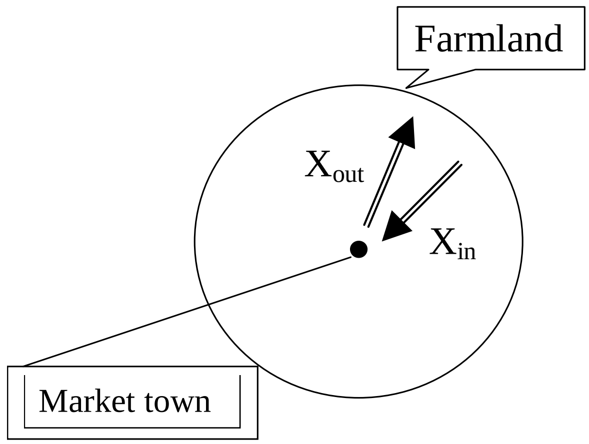

Figure 5: Geography of the Environment

The structure of equilibria in this closed model (economy with production) resembles that in the pure exchange economy.

In the “closed” environment, the set of equilibria $\Gamma^{cm}(c)$ is:

{0} if c c Ɥcis:

$\Gamma^{cm}(c) =$

Depending on market structure[26], the portion of the surplus that farmers obtain should be in the [0, D (γ, G

Hence, in environments with strong non-market systems of production and exchange or with strong social networks – each of which can be interpreted as high levels of ψ – the welfare-improving role of a Minimal State is diminished.

It is important to stress that the role of the "state" in this model does not necessarily have to be played by the o cial government. The only implication that we can draw from the analysis in this respect is that the emergence of trade (market economy) gives rise to the institution of property rights. If protection is not (adequately) provided by o punish detected robbers and re-instate property to its owners. By so doing it imposes direct or indirect costs on the robbers, thus encouraging more agents to trade fairly. Mutually beneficial trades occur more often, and more is produced and consumed.

which agents have an option may in

Let

$$F(y;c) = \kappa(y)\,[V_t^g(y;c) \quad V_r^g(y;c)]\,; \tag{19}$$

provided $V_r^g > 0$. The last inequality stems from the fact that $V_t^g(0;c) - V_r^g(0;c) < 0$ (by assumption) and $V_t^g(0;c) = 0$ by definition.

Clearly, if $V_t^g(\gamma;c) - V_r^g(\gamma;c) < 0$ for a range of $\gamma : \gamma \notin 0$, then none of the values in the range is consistent with a stationary subgame perfect Nash equilibrium. Similarly $V_t^g(1;c) - V_r^g(1;c) > 0$ implies $\gamma = 1$ is an equilibrium, as the one shot deviation (rob and then trade) is unprofitable:

$$\tilde{V}_t^g(1;c) - V_t^g(1;c) < 0, \tag{22}$$

where

$$\tilde{V}_t^g(1;c) = \beta\left(G(1-\omega) - c\omega + \delta V_t^g(1;c)\right) + (1-\beta)\,\delta V_t^g(1;c). \tag{23}$$

Indeed,

$$\begin{aligned} & \tilde{V}_t^g(1;c) - V_t^g(1;c) = \\ = \; & G\beta - G\beta\omega - c\beta\omega - V_t^g(1;c)(1-\delta) < \\ < \; & G\beta - G\beta\omega - c\beta\omega - V_r^g(1;c)(1-\delta) = 0 \end{aligned} \tag{24}$$

■

If $c \geq \underline{c}(\omega)$, then $\gamma = 1$ is an equilibrium.

Given $\gamma = 1$, the value of trading forever is

$$V_t^g(1;c) = \beta\frac{G}{1-\delta+\beta\delta}, \tag{25}$$

while the value of robbing forever is

$$V_r^g(1;c) = \beta\frac{(G(1-\omega) - c\omega)}{1-\delta}. \tag{26}$$

Clearly, if $F(1;c) = V_t^g(1;c) - V_r^g(1;c) = 0$, then $\gamma = 1$ is an equilibrium. If $F(1;c) > 0$ then $\gamma = 1$ is an equilibrium by lemma 13. But $F(1;c) > 0$ if and only if

$$c > \underline{c}(\omega) \equiv \frac{G(\beta\delta(1-\omega) - \omega(1-\delta))}{\omega(\beta\delta+1-\delta)}. \tag{27}$$

■

If $c \quad \bar{c}(\omega)$, then $\gamma = 0$ is an equilibrium.

The value of trading forever provided $\gamma = 0$ is zero. Thus, for $F(0; c) \quad 0$, it is su cient to have the value of robbing forever to be positive positive, $V_r^g(0; c) \quad 0$, which is equivalent to requiring

$$G(1 \quad \omega) \quad c\omega \quad 0, \tag{28}$$

so that

$$c \quad \bar{c}(\omega) \quad \frac{G(1 \quad \omega)}{\omega}. \tag{29}$$

The conclusion then follows, again, by lemma 13. ■

If $c > \bar{c}(\omega)$ there is a unique equilibrium $\gamma = 1$.

If

$\gamma_L(c)$ is unstable and $\gamma_H(c)$ is stable. [30]

Fix c_0 in the range in which γ_L is well defined. Assume that the fraction of fair traders is $\gamma_L(c_0)$. Let $c_1 = c_0 + \varepsilon$, $\varepsilon > 0$. Then $F(\gamma_L(c_0), c_1) > 0$, so that the fraction of the fair traders should increase.[31] It will continue to grow till it reaches $\gamma_H(c_1)$ as $\gamma_L(c_1) < \gamma_L(c_0) < \gamma_H(c_1)$ and $F(\gamma_H(c_1), c_1) = 0$. $F(\gamma, c_1) < 0$ for $\gamma > \gamma_H(c_1)$,[32] so agents should gravitate towards $\gamma_H(c_1)$ also. If the punishment is reduced even marginally at $\gamma_L(c_0)$, then $F(\gamma_L(c_0), c_1) < 0$, thus robbing becomes more attractive. As the last inequality is satisfied for all $\gamma < \gamma_L(c_0)$, the new equilibrium will be $\gamma = 0$. Thus, slightly more punishment will increase the proportion that $\gamma = 1$ is an equilibrium and $F(0; c) < 0$ implies $\gamma = 0$ is an equilibrium. Recall representation (20). Clearly, $k_F < 0$, if

$$c < \bar{c}(\omega) = G\frac{(1 \quad \omega)}{\omega}, \tag{30}$$

while $b_F > 0$, and $a_F < 0$ if

$$c < \frac{G(1 \quad \omega^2)}{\omega^2}. \tag{31}$$

The polynomial is maximized at $\gamma = \gamma$, where

$$\gamma \quad = \frac{1}{2}\frac{(G(1 \quad \delta) + G\omega(1 \quad \delta) + c\omega(1 \quad \delta + \beta\delta))}{(\quad G\omega^2 + G \quad c\omega^2)\,\beta\delta}. \tag{32}$$

If $\gamma \quad > 1$, then $F(1; c) > 0$, as the upper root should be above unity. Evidently γ > 1 if and only if

$$c > c \qquad \frac{(2\beta\delta(1 \quad \omega) \quad (1 \quad \delta))}{(\beta\delta \quad \delta + 2\beta\delta\omega + 1)}\frac{(\omega + 1)\,G}{\omega}. \tag{33}$$

To derive the lower bound, note that there are two possible cases that can lead the polynomial $F(\gamma; c)$ to be negative for all $\gamma \quad [0, 1]$. The first case occurs when γ , at which F is maximized, is above unity. In this case F hits zero at most once between zero and one. Thus, if $F(1; c) < 0$, then it is negative for any $\gamma \quad [0, 1]$. Secondly, if $\gamma \quad < 1$ and $F(\gamma \; ; c) < 0$, then, $F(\gamma; c) < 0$ for any γ. We will start with the first case, as it generates a higher lower bound on c, given that γ strictly increases in c (which can be verified directly from (32)).

If $\beta\delta + \delta \quad 1$ and $c < \underline{c}(\omega)$, then there is a unique equilibrium $\gamma = 0$.

■

Note that if $c < \underline{c}(\omega)$, then $F(1; c) < 0$. If $c > c$, then $\gamma \quad > 1$. Therefore if $c \quad < \underline{c}(\omega)$, then $c \quad [c \;, \underline{c}(\omega$ $ + fG6ð as required. It is left to show that in this case if $c < c$, then the equilibrium remains unique, $\gamma = 0$. Consider $c = c$. As $c < \underline{c}(\omega)$, and $\gamma = 1$, it implies $F(\gamma \;\; (c);c) < 0$. As γ is the maximand of F, it follows that $F(\gamma;c) < 0$ for any γ. Now consider $c_0 < c$. It can be easily shown that F decreases in c for any γ. Therefore, $F(\gamma;c_0) < 0$. The case of equality $\beta\delta + \delta = 1$ is trivial. This completes the proof of proposition (2).

If $\beta\delta + \delta > 1$ and $c < \underline{\underline{c}}(\omega)$, then there is a unique equilibrium $\gamma = 0$.

■

If $\beta\delta + \delta > 1$ then $c \;\; > \underline{c}(\omega)$, therefore, for $c < \underline{c}(\omega) < c$ first, $F(1;c) < 0$ and, second, $\gamma \;\; < 1$. Therefore, the parabola $F(\gamma;c)$ can cross zero twice if the discriminant

$$H(c;\beta,\delta,\omega) \quad b_F^2(c) \quad 4a_F(c)\,k_F(c) \tag{36}$$

is positive. Whenever H is negative, $F(\gamma;c)$ lies below zero for any γ and, in this case, the only equilibrium is $\gamma = 0$. It remains to derive lower bound, $\underline{c}(\omega)$, on the severity of punishment that assures that $H(c;\beta,\delta,\omega) < 0$. Clearly, $H(c;\beta,\delta,\omega)$ is quadratic in c :

$$H(c;\beta,\delta,\omega) = c^2 a_H(\beta) + c b_H(\beta) + k_H(\beta), \tag{37}$$

where

$$k_H(\beta) = (G \quad G\delta + G\omega \quad G\delta\omega)^2 \tag{38}$$

$$4G\beta\delta(1 \quad \omega) \;\; G\omega^2 \quad G \;\; (\delta \quad \beta\delta + \beta\delta\omega \quad 1); \tag{39}$$

$$b_H(\beta) = G \quad \begin{matrix} 2(\omega \quad \delta\omega + \beta\delta\omega)(\omega \quad \delta\omega + 1 \quad \delta) + \\ +4\beta\delta(\beta\delta(1 \quad \omega) + 1 \quad \delta)\,\omega(2\omega + 1)(1 \quad \omega) \end{matrix} \quad ; \tag{40}$$

$$a_H(\beta) = \omega^2(\delta \quad \beta\delta + 2\beta\delta\omega \quad 1)^2 \tag{41}$$

Since $a_H(\beta) > 0, b_H(\beta) >$

turn, $k_H(\beta)$ is quadratic in β :

$$
\begin{aligned}
k_H(\beta) &= a_k(\delta)\beta^2 + b_k(\delta)\beta + k_k(\delta)\,, && (42)\\
k_k(\delta) &= G(1 \quad \delta + \omega \quad \delta\omega)^2\,; && (43)\\
b_k(\delta) &= 4G^2\delta(1 \quad \delta)(1 \quad \omega) \;\; \omega^2 \quad 1 \;\;; && (44)\\
a_k(\delta) &= 4G^2\delta^2(1 \quad \omega)^2 \;\; \omega^2 \quad 1 \;\;, && (45)
\end{aligned}
$$

Observe that $k_k(\delta) > 0, b_k(\delta) < 0, a_k(\delta) < 0$. Thus, the polynomial $k_H(\beta)$ has two roots, as long as $k_k(\delta), b_k(\delta), a_k(\delta) \neq 0$, . The lower root is negative, while the upper one,

$$\beta_H = \frac{b_k(\delta) \qquad \sqrt{(b_k(\delta))^2 \quad 4a_k(\delta)\,k_k(\delta)}}{2a_k(\delta)} \tag{46}$$

$$\beta_H = \beta_1(\omega) \quad \frac{(1 \quad \delta)((\omega \quad 1)^2 \qquad \sqrt{2(1 \quad \omega)^3})}{2\delta(\omega \quad 1)^3} \tag{47}$$

is positive. It is also below unity as long as

$$\delta > \delta_1(\omega) \quad \frac{\sqrt{2(1 \quad \omega)} \quad (1 \quad \omega)}{\sqrt{2(1 \quad \omega)} + (1 \quad 2\omega)(1 \quad \omega)} \tag{48}$$

For $\delta > \delta_1(\omega)$ and $\beta > \beta_1(\omega)$, $k_H(\beta) < 0$. In this case polynomial $H(c;\beta,\delta,\omega)$ has two roots of opposing sign. Recall that $a_H(\beta) > 0$, so that $H(c;\beta,\delta,\omega)$ is negative for all the values of c in between the two roots, which means F does not have real roots (γ) and is always negative implying that the only equilibrium is $\gamma = 0$. If c is below the lower root of H (c), quadratic polynomial F (γ) has negative roots. Thus as long as c is below the upper (positive) root of $H(c;\beta,\delta,\omega)$, the only equilibrium is $\gamma = 0$. Denote this root by $\underline{\underline{c}}(\omega)$:

$$\underline{\underline{c}}(\omega) \quad \frac{b_H + \sqrt{b_H^2 \quad 4a_H k_H}}{2a_H}. \tag{49}$$

Finally, assume that $\beta\delta + \delta > 1$ and

$$\underline{\underline{c}}(\omega) < c < \underline{c}(\omega)$$

We have to show that in this case there are three equilibria: $\gamma = 0$, and a couple $\gamma_L < \gamma_H < 1$. The two roots of the polynomial $F(\gamma; c)$, are

$$\gamma_L(c) = \quad b_F$$

that $D(\gamma, G)$ is increasing in the proportion of fair traders, γ. The latter stems from the fact that

$$\frac{\partial D(\gamma, G)}{\partial \gamma} = \frac{G\beta(\beta\delta \quad \beta\delta\omega + 1 \quad \delta)}{(\beta\delta \quad \delta \quad \beta\delta\omega + \beta\gamma\delta\omega + 1)^2} > 0 \tag{54}$$

and

$$K'(G) = \frac{d}{dG}\left(\frac{b_F^2(G)}{|a_F(G)|^2} + \frac{4k_F(G)}{|a_F(G)|}\right) = \tag{63}$$

$$= 2T(G)\left(\frac{(\omega+1)N(G)}{\delta\beta\left(G(1-\omega^2)-c\omega^2\right)} + 2(1-\omega)\right) < 0, \tag{64}$$

where

$$N(G) = G(1-\delta) + G\omega(1-\delta) + c\omega(1-\delta) + c\beta\delta\omega, \tag{65}$$

$$T(G) = \frac{c\omega(\delta-\beta\delta+\beta\delta\omega-1)}{\delta\beta\left(G(1-\omega^2)-c\omega^2\right)^2}. \tag{66}$$

Then

$$\gamma_H'(G) = \gamma'(G) + \frac{K'(G)}{\sqrt{K(G)}} < 0 \tag{67}$$

Then

$$\frac{\partial D(\gamma, G)}{\partial\gamma}\gamma_H'(G) < 0 \tag{68}$$

It has been shown that γ_H is increasing in c, thus the claim follows. ■

6 First, we have to assume that $c > 0$. The value of trading forever and robbing forever in this environment, correspondingly are

$$V_t^{cg}(\gamma) = \beta \frac{\gamma G}{1 \quad \delta(1 \quad \beta)} \tag{69}$$

$$V_r^{cg}(\gamma; c) = \frac{\beta(\gamma + 1)((1 \quad \omega)G \quad \omega c)}{2(1 \quad \delta) + \beta\delta(1 \quad \gamma)}. \tag{70}$$

As before, if $c > \bar{c}(\omega)$, we have

$$V_r^{cg} < 0 \quad V_t^{cg}, \tag{71}$$

so the only equilibrium is $\gamma = 1$. Moreover, this is an equilibrium as long as $V_t^{cg}(1)$ $V_r^{cg}(1; c) \quad 0$, which is equivalent to setting, as before,

$$c > \underline{c}(\omega) = \frac{(\beta\delta(1 \quad \omega) \quad \omega(1 \quad \delta))G}{(\beta\delta \quad \delta + 1)\omega} \tag{72}$$

Now assume $c < \bar{c}(\omega) = G(1 \quad \omega)/\omega$.

The di erence between the two value from trading fairly, $V^{cg}(\text{trade})$, and that from robbing,$V^{cg}(\text{rob})$, should be equal to zero in the equilibrium. This is equivalent to requiring that $F^{cg}(\gamma, c) = 0$, where

$$F^{cg}(\gamma, c) \quad (2(1 \quad \delta) + \beta\delta(1 \quad \gamma))\gamma G \quad (\gamma + 1)((1 \quad \omega)G \quad \omega c)(1 \quad \delta(1 \quad \beta)) \tag{73}$$

Again, $F^{cg}(\gamma, c)$ is a quadratic polynomial in γ :

$F^{cg}(\gamma, c) = k\gamma$ [illegible]

$k_1(c) < 0, a_1(c) < 0$. If $b_1(c) < 0$, then $F^{cg}(y; c) < 0$ for all $y > 0$. Thus, equilibrium is $y = 0$.

If $b_1(c)$

by $\hat{\gamma}$,

$$\hat{\gamma} = \frac{G}{\omega\,(G + c)} \quad 1. \tag{90}$$

This root, $\hat{\gamma}$, is positive i

$$c < \frac{G\,(1 \quad \omega)}{\omega} = \overline{c}\,(\omega) \tag{91}$$

which is consistent with the assumptions in the statement of the proposition, as $c < \underline{c}\,(\omega) < \overline{c}\,(\omega)$. Observe that $F^{cg}(\gamma, c)$ is negative if $c < \underline{c}\,(\omega)$. Indeed,

$$F^{cg}(\gamma, c) = \frac{(G\omega + c\omega \quad G\beta\delta \quad G\delta\omega \quad c\delta\omega + G\beta\delta\omega + c\beta\delta\omega)}{(G + c)^2\,\omega^2}\,(G(1 \quad \omega) \quad c\omega)\,G \tag{92}$$

Provided $(G(1 \quad \omega) \quad c\omega) > 0$, so that $c < \overline{c}\,(\omega)$,

$$F^{cg}(\gamma, c) \quad 0 \tag{93}$$

if

$$G\omega + c\omega \quad G\beta\delta \quad G\delta\omega \quad c\delta\omega + G\beta\delta\omega + c\beta\delta\omega \quad 0 \tag{94}$$

The last inequality holds i

$$c \quad \frac{(\beta\delta(1 \quad \omega) \quad \omega(1 \quad \delta))\,G}{(\beta\delta \quad \delta + 1)\,\omega} = \underline{c}\,(\omega)\,, \tag{95}$$

It follows that the lower root of $F^{cg}(\gamma, c)$ should be below the lower root of $F(\gamma, c)$ and the opposite is true for the upper root, i.e., $\gamma_H^{cg}(c) > \gamma_H(c)$, which corresponds to the claim in the proposition.

Second, consider the complementary case to (96), $c \quad \hat{c}(\omega)$. The goal is to show that neither F nor F^{cg} have no positive roots in this range, in other words, $\hat{c}(\omega) <$ max $\underline{\underline{c}}(\omega), \underline{\underline{c}}^{cg}(\omega)$. Indeed, take $c = \hat{c}(\omega)$, then the only intersection point of F and F^{cg} is $\hat{\gamma} = 1$:

$$F^{x}(\gamma, \hat{c}) = \quad \frac{1}{2} \quad G(\gamma \quad 1)^2. \tag{97}$$

It follows that $F^{cg}(\gamma, \hat{c}) \quad F(\gamma, \hat{c})$ for any γ and the two are equal and tangent at $\gamma = 1$.

Moreover, the polynomial

$$F(\gamma, \hat{c}) = \frac{1}{2}G(\delta \quad \beta\delta + \beta\delta\omega \quad 1) + \frac{1}{2}G\gamma^2(\beta\delta\omega \quad 2\beta\delta) + \frac{1}{2}G\gamma(\beta\delta \quad 3\delta \quad 2\beta\delta\omega + 3) \tag{98}$$

has only negative roots, and therefore, is negative for $\gamma \quad [0, 1]$. Indeed, its discriminant,

1

Clearly, $\varphi(c)$ is continuous in c for $c > \quad G$. But the range of punishments we are considering falls into this category, as $\hat{c} = \frac{G(1 \quad 2\omega)}{2\omega} > G$, clearly. (Besides, if $c + G \quad 0$, a thief will never pay a bribe.) Moreover,

$$\varphi(\underline{c}(\omega)) = (\beta\delta \quad \delta \quad \beta\delta\omega + 1)(\delta + \beta\delta$$

9 In this case the equilibrium demand for government is

$$
\begin{aligned}
D^{cg}(\gamma, G) &= \beta \frac{G\gamma}{1 \quad \delta + \beta\delta} \quad \frac{\beta G}{\beta\delta \quad 2\delta + 2} = \qquad (109) \\
&= \frac{(2\gamma + \delta \quad \beta\delta \quad 2\gamma\delta + \beta\gamma\delta \quad 1)}{(\beta\delta \quad 2\delta + 2)(\beta\delta \quad \delta + 1)} G\beta,
\end{aligned}
$$

which is, again proportional to the gains from trade and is positive (an increasing in G) i

$$\gamma > \underline{\gamma}^{gc} \quad \frac{1 \quad \delta + \beta\delta}{2 \quad 2\delta + \beta\delta}. \qquad (110)$$

Note that the lower bound on the equilibrium γ now is higher that under non-corrupt government (for strictly positive ω), $\underline{\gamma}^{gc} > \underline{\gamma}$.

Therefore, in the presence of corruption, again the upper equilibrium $\gamma_H^{cg}(G)$ is decreasing in G. ■

Assume $\beta\delta + \delta > 1$ and c [$\underline{\underline{c}}(\omega)$, $\underline{c}(\omega)$), so that the protection agency can induce equilibrium $\gamma = \gamma_H$

The derivative of the above expression with respect to δ is

$$\frac{\beta^2 (1 - \delta + \beta\delta)^2 + 2\beta^2\delta (1 - \beta)(1 - \delta + \beta\delta)}{(1 - \delta + \beta\delta)^4} > 0 \tag{121}$$

so it is maximized when $\delta = 1$. Hence,

$$c^{\#} < G\left(\frac{\beta^2}{\beta^2} - 1\right) = 0 \tag{122}$$

So $c^{\#}$ has to be negative.

This implies that the threshold $c^{\#} < 0$. As by assumption, $c \geq \underline{c} > 0$, we conclude that $D(\gamma, G) - D^{cg}(\gamma, G) > 0$. ■

10 In this environment, the population of Market town is

Suppose $c < \bar{c}^{cm}(\omega)$. Then, an equilibrium exists in which there is no trade: $\gamma = 0$.

23 Let H be the value function of being in Farmland.

$$H = \delta \max\{H, V\} \quad H = \max\{0, \delta V\} \tag{125}$$

In the case of anarchy, in a steady state,

$$V^a = \beta \max \quad \gamma G + \delta H, \gamma W^a + (1 \quad \gamma)\frac{1}{2}(W^a + \delta H) \tag{126}$$

$$+ (1 \quad \beta)\delta V^a \tag{127}$$

If the agent chooses to trade this period, her value function is defined by

$$V^a(\text{trade}) = \beta(\gamma G + \delta H) + (1 \quad \beta)\delta V^a \tag{128}$$

If there are any equilibria other than the 'Ghost Town' equilibrium, it must be that $V^a \quad 0$, so that this becomes

$$V^a(\text{trade}) = \beta \quad \gamma G + \delta^2 V^a \quad + (1 \quad \beta)\delta V^a \tag{129}$$

On the other hand, if she decides to rob her value function becomes

$$V^a(\text{rob}) = \beta \quad \gamma W^a + (1 \quad \gamma)\frac{1}{2}(W^a + \delta F) \quad + (1 \quad \beta)\delta V^a \tag{130}$$

By comparing the contemporaneous terms of equations (129) and (130) ,for any internal values of the parameters, $\beta, \delta \quad (0, 1)$, and regardless of continuation values and strategies,

$$V^a(\text{rob}) > V^a(\text{trade})2s$$

as good to steal as to trade.

$$V^{g} \quad \frac{[\omega c \quad (1 \quad \omega)G]}{\delta(1 \quad \delta)(1 \quad \omega)} \tag{141}$$

The value function must equal the value of stealing, so that

$$V^{g} = \beta \frac{[(1 \quad \omega)G \quad \omega c]}{(1 \quad \delta)[2 + \delta\beta(1 \quad \omega)]} \tag{142}$$

Thus, the corresponding parameter restriction is

$$\beta[(1 \quad \omega)G \quad \omega c]\,\delta(1 \quad \delta)(1 \quad \omega) \quad (1 \quad \delta)[2 + \delta\beta(1 \quad \omega)][\omega c \quad (1 \quad \omega)G] \tag{143}$$

Now there are thre[illegible]. First, suppose th0.67006

■

11 Suppose y

leaves several possible cases of F^{cm}. First, suppose $a_{cm}(\omega) > 0$. Then, it must be that

$$c > \frac{(1 \quad \omega^2)\,G}{\omega^2} > \frac{(1 \quad \omega)\,G}{\omega} = \bar{c}^{cm}(\omega) \qquad (16167912335ci8252$$

$F^{cm}(1;c) \quad 0$, which implies that

$$c \quad \frac{\beta\delta(1 \quad \omega) \quad \omega}{(1+\beta\delta)\,\omega} G = \underline{c}^{cm}(\omega) \tag{167}$$

However, this contradicts the initial assumption that $c \quad \overline{c}(\omega) > \underline{c}^{cm}(\omega)$. Hence this type of equilibrium does not exist, so that if

$$c \quad \frac{(1 \quad \omega)G}{\omega}, \frac{(1 \quad \omega^2)\,G}{\omega^2} \tag{168}$$

then there is a unique equilibrium in which $\gamma = 1$ also. Our results so far are that if $c > \overline{c}(\omega)$, there is a unique equilibrium in which $\gamma = 1$.

In the second case, $k_{cm}(c) < 0$:

$$c < \frac{(1 \quad \omega)G}{\omega} \tag{169}$$

so that there does exist an equilibrium with $\gamma = 0$. In addition, we know that, generically, $F^{cm}(\gamma; c)$ will have two positive real roots or none in this region of parameter space, since $k_{cm}(c) < 0$. Let

$$\gamma_{cm} = \arg\max_{\gamma} F^{cm}(\gamma; c) \tag{170}$$

The equation for γ_{cm} is

$$\begin{aligned} \frac{\partial F^{cm}(\gamma; c)}{\partial \gamma} &= 2\gamma_{cm} a_{cm}(c) + b_{cm}(c) = 0 \qquad (171) \\ \gamma_{cm} &= \frac{(1+\omega)\,G + \omega c + \beta\delta\omega c}{2} \end{aligned}$$

equilibrium. How to show this? First,

$$F^{cm}(1;c) \quad = \quad \beta\delta\omega^2 c + \omega c + \beta\delta\omega c + \omega c + \beta\delta(1 \quad \omega)\omega c \tag{174}$$

$$\beta\delta \;\; 1 \quad \omega^2 \;\; G + 2\omega G \quad \beta\delta(1 \quad \omega)(1 \quad \omega)G \tag{175}$$

$$F^{cm}(1;c) = 2[1 + \beta\delta(1 \quad \delta)]\omega c \quad 2\beta\delta(1 \quad \delta)(1 \quad \omega)G + 2\omega G \tag{176}$$

$F^{cm}(1,c)$ is positive i

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