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Comparison of Urban Equilibria: The Conflict Between Efficiency and Equity in the Choice of the Central City's Majority

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COMPARISON OF URBAN EQUILIBRIA:
THE CONFLICT BETWEEN EFFICIENCY AND EQUITY
IN THE CHOICE OF THE CENTRAL CITY'S MAJORITY

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ABSTRACT

A circular metropolitan area consists of a central city and a suburbs. Households sort over the two jurisdictions based on public service levels and their costs of commuting to the metropolitan center. Using numerical simulations, we show (1) there typically exist two equilibria: one in which the poor form the voting majority in the central city and the other in which the rich form the majority in the central city; (2) there is an efficiency *v.* equity trade-off as to which equilibrium is preferred; (3) if the central city contains only poor households, equity favors expanding the central city to include rich households; and (4) as a metropolitan population grows, it is likely to select the equilibrium in which poor households form the city's majority.

JEL Classification: H73, R12, R14.

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COMPARISON OF URBAN EQUILIBRIA:

1. INTRODUCTION

In Tiebout's (1956) seminal model of fiscal competition, households sort themselves between communities based on their benefits from the public service, and the resulting allocation of households to communities is first-best efficient.[1] In Tiebout's model, the public service is financed by a residency tax and community boundaries are flexible. Hamilton (1975, 1976) shows that, if the residency tax is replaced by a property tax, zoning can allow households to sort across local jurisdictions to achieve first-best efficiency. Alternatively, Elickson (1971), Yinger (1982), Epple, Filimon and Romer (1984, 1993) and Epple and Romer (1991) describe communities as having fixed boundaries and a property tax, but no zoning powers. In such models, the public service and the property tax are capitalized into the land price in a jurisdiction, distorting the location and housing decisions of individual households.[2] In general, the equilibrium outcomes in such economies are not first-best efficient.

In contrast to Tiebout's model, Mills (1967) and Muth (1969) present a spatial model of the monocentric city: income sorting occurs from the interaction of commuting costs and land demand. If land demand is sufficiently income elastic, the saving achieved by the purchase of land further from the city's center is greater for the rich households and compensates them for the associated increase in commuting cost.[3] In this case poor households win the bid for land near the city's center and vote for low levels of public services. Mills (1967), Muth (1969), and Mills and Lubuele (1997) suggest that this sorting mechanism is a key factor in the concentration of the poor into U.S. central cities and in the resulting low level of public-service quality in those cities. Conversely, if commuting costs increase with income and if land demand is unresponsive to income changes, rich households outbid poor households for locations closer to the city's center. Wheaton (1977) and Glaeser, Kahn, and Rappaport (2000) find evidence that the income elasticity of land demand is quite small and as a result conclude that this type of sorting cannot play a major role in explaining the centralization of the poor.

In general, households choose communities based on both public services and commuting costs. We consider a stylized model in which the central city has an exogenous boundary and is surrounded by the suburbs. All households must commute to the central business district which is located at the center of the city. The model has two income-classes. Rich households have higher commuting costs per mile than poor households and land demand is relatively income inelastic; *ceteris paribus* rich households outbid poor households for land near the city center. The public service in each community is determined by voting. We find multiple equilibria in which each income class controls one jurisdiction. In one equilibrium, poor households are the majority in the central city, voting low public services in the city; in the second equilibrium, rich households are the majority in the central city, voting high public services there.

Which equilibrium is preferred? If the city boundary is free to adjust until each jurisdiction contains only one income class, efficiency is clearly higher in the equilibrium in which rich households reside in the city: there is perfect matching of households with their desired public service and commuting costs are minimized. From an equity standpoint, however, poor households obtain the most utility if they form the majority in the city and if the city is large enough to have vacant land at its jurisdictional boundary - this arrangement decreases competition for urban land and thereby lowers the rent that the poor must pay for living in the city.

If the city boundary is fixed, the form of the efficient equilibrium depends on the city size, and the efficiency-equity trade-off continues to be present. For example, if the size of the cental city is small so that the city contains one income class and the suburbs contains both income classes, efficiency is higher in the equilibrium in which the city contains only poor households and the suburbs contains poor and rich households, with rich households forming the suburban majority. This suggests that getting a good match of rich households with high public service levels is more important than trying to reduce the resources spent on commuting. In contrast, equity considerations favor the equilibrium in which the city contains only rich households, because this arrangement enables the poor households to get their preferred public service at a lower rent.

The jurisdictional boundary between the central city and the suburbs is neither chosen by the market nor is it fixed; it is a variable which is potentially chosen by the policy-maker. For example, in 1993 Memphis (USA) proposed merging with its near suburbs and a similar exercise was completed by Toronto (Canada) in 1998. One motivation for such a change is strategic: by merging with its suburbs a central city is able to enlarge its commercial tax base and reduce tax competition. We consider another, perhaps more fundamental, feature of boundary changes: the change in the sorting of households by income between the city and suburbs induces a change in rents. Our simulations suggest that, if the status-quo is the equilibrium in which poor households congregate in the central city, expanding the city so that it contains some rich households benefits poor households. In our model, taxes are residency taxes so that this benefit to poor households does not arise because rich households pay more taxes; poor households benefit because their rents fall.

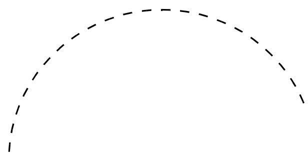
Which of the two possible equilibria - poor households or rich households forming the central city's majority - is more likely? We consider the comparative statics of an increase in the metropolitan population in the presence of a fixed city boundary. When the population is small,

all households live in the central city; there is a majority of poor households and the public service is low. As the population increases, city rents increase and the edge of urban

2. THE MODEL

A household *h* lives in a community and obtains utility *U*

The community provides the public service g .[5]



As the location moves across the jurisdictional boundary, the public service changes from the level desired by the rich households to the level desired by poor households: poor suburban households are willing to pay the premium *FD* to move across the jurisdictional boundary. This premium ensures that the rent exceeds the reservation rent r_0 and there is no undeveloped land. The rent schedule in the city is *DE* and along *DE* the rent rises at rate tM_1/a_1. Rich households do not live in the city because the rent exceeds their willingness to pay for the lower public service.

In Figure 2(b), the city boundary has expanded to include all poor households and some rich households. There are no poor households in the suburbs. The rent at the suburban fringe is and rises at ra *tM2/a2* BC along the location moves towards the city center. As the location crosses the jurisdictional boundary, the rent would have to fall by *CH* (and become negative in this example) if a rich household were to be willing to live on the city side of the boundary. *HEG* is the bid-rent curve of a rich household in the city.

There is undeveloped land in the city and the poor households at the limit of development pay rent *r ;rent rises at rate tM1*/a1 along *DE*. At distance *x* from the city center, rent equals the willingness to pay of a rich suburban household to live at *x*, and rich households live at the

The population and average lot size are chosen to be close to the values observed in the population. The lot size for poor and rich households is adjusted down and up from the average lot size, respectively, in order to be consistent with a 0.3 income elasticity of demand for land - an elasticity value which is consistent with the recent estimates by Glaeser, Kahn, and Rappaport (1999).[12] The population and housing demand parameters imply that the area of developed land in the metropolitan area is 90,000 acres: this translates to a radius (*Y*) of 6.69 miles if the city has no vacant land.

The values for D and * are chosen so that the price and income elasticities of demand for the public service are -0.5 and 0.7, respectively, which is consistent with current estimates in the literature (see Ross and Yinger (1999)). These parameter values (and $A=3$) imply that poor households vote a public service level of 1452 ($ per year) and rich households vote a public service level of 3132 ($ per year). The "average" calculated value for the share of income spent on public service is therefore (1452+3132)/(1500 + 4500) = .076 [13]; in contrast, the societal value is 0.13.[14] When we raised the value of the parameter *A* so that the simulated value lay closer to the observed value, there was little effect on the results except that we ceased to find an equilibrium configuration in which both poor and rich households live in the city and in which all city land is developed.[15] In order to illustrate an equilibrium with this form, we decided to use the lower value of *A*.

Our focus is on comparing equilibria and what is important is the relative value of rents. Therefore, for convenience, we set the reservation rent to zero.

The simulations of this section investigate how the equilibrium outcomes change as the city's jurisdictional boundary, *B*, is increased from 0.5 miles to 6.5 miles in increments of 1 mile. For each value of *B*, we found two equilibria: an equilibrium in which poor households are the majority in the city and rich households are the majority in the suburbs *and* an equilibrium in which the majorities are reversed. The former configurations are grouped together as Case 1, the latter configurations are grouped together as Case 2.[16]

3.2 Case 1: Poor are majority in city, rich are majority in suburbs

Table 2 presents the simulation results for the equilibrium in which poor households are the city's majority and rich households are the suburban majority. This might be considered the common U.S. outcome, with high levels of public services in the suburbs. The first row shows the city's jurisdictional boundary (assigned exogenously). The second row shows the equilibrium case (using labels which are to be discussed) which arises for the given boundary size. The next row shows the average rent;[17] the next two rows show the utility achieved by poor and rich households. The next four rows show some characteristics of the metropolitan area; the number of poor and rich households living in the city is shown in Table 4. The final two rows show the utility change a poor and rich household would experience if he were to move from the community in which he resides to the other community.

Metropolitan population, $N = 270{,}000$

Attributes of metropolitan area	Symbol	Equilibrium values						
City jurisdictional boundary (miles from center)	B	0.5	1.5	2.5	3.5	4.5	5.5	6.5
Equilibrium Case Number (label from text)		1.1	1.1	1.1	1.1	1.2	1.3	1.3
Average rent per household ($ per year)		756	783	852	969	558	398	413
Utility of poor households ($ per year)		9,941	9,941	9,941	9,941	11,006	11,158	11,088
Utility of rich households ($ per year)		35,465	35,405	35,293	35,136	34,973	34,761	34,451
Boundary between income groups in city (miles)	x	0	0	0	0	1.11	2.46	3.15
Boundary of city development (miles)	X	0.5	1.5	2.5	3.5	4.5	5.01	5.38
Boundary between income groups in suburbs (miles)	y	5.10	5.29	5.66	6.16	6.69	7.07	7.62
Boundary of suburban development (miles)	Y	6.69	6.69	6.69	6.69	6.69	7.07	7.62
Poor household's utility change on moving from city to suburbs ($/yr)		0	0	0	0	-1,065	-1,288	-1,321
Rich household's utility change on moving from suburbs to city ($/yr)		-1,591	-1,531	-1,419	-1,262	0	0	0

Table 2: Simulation results for Case 1

The rent paid at the suburban fringe, r_0, anchors the utility achieved by poor households, and their utility does not change as the boundary expands. The efficiency gain (from better matching of poor households with their desired public service) is captured as rent by landlords. Because rich households are being pushed away from the center and have high commuting costs, their utility falls.

As the jurisdictional boundary moves outwards from 3.5 to 4.5 (miles), the equilibrium structure changes to the form of Case 1.2. The city has grown sufficiently large that it can contain all poor households and some rich households; the remaining rich households live in the suburbs and there is no undeveloped city land. The level of the rent schedule in the city is now determined by the rent required to make a rich household indifferent between the communities. Rich households in the city need to be compensated for the lower public service, and average rents fall accordingly. The utility of poor households is no longer determined by what happens at the suburban edge; the lower city rent raises their utility. The utility of rich households is determined by the utility they obtain in the suburbs: their rent plus commuting costs has increased and their utility falls.

When B moves from 4.5 to 5.5 (miles), vacant land appears near the city boundary, and continues to be determined by what happens at the suburban edge: their commuting costs are increasing and their utility falls.

Finally, when *B* increases from 5.5 to 6.5 (miles), more rich households migrate into the city to avoid the longer commute; this pushes up the rent in the city at the locations occupied by rich households. The rent of poor households is anchored by the reservation price of land.. Poor households are being pushed further out and, with the rent at the edge of development fixed, their utility falls. Rich households in the suburbs continue to incur higher commuting costs and the utility of rich households, determined by what happens in the suburbs, continues to decrease.

3.3 Case 2: Rich are majority in city; poor are majority in suburbs

Table 3 presents the simulation results for the equilibrium in which rich households are the city's majority and poor households are the suburban majority. When the jurisdictional boundary *B* is set to values between 0.5 and 4.5 (miles), Case 2.1 arises: the city contains only rich households but it is so small that it cannot contain all the rich households so that some members of this group reside in the suburbs with the poor households; there is no undeveloped city land.

Metropolitan Population, *N = 270,000*

Attributes of metropolitan area	Symbol	Equilibrium values						
City jurisdictional boundary (miles from center)	*B*	0.5	1.5	2.5	3.5	4.5	5.5	6.5
Equilibrium Case Number (label from text)		2.1	2.1	2.1	2.1	2.1	2.2	2.2
Average rent per household ($ per year)		762	838	989	1,216	1,518	544	536
Utility of poor households ($ per year)		10,843	10,843	10,843	10,843	10,843	10,781	10,629
Utility of rich households ($ per year)		33,527	33,527	33,527	33,527	33,527	35,883	35,883
Boundary between income groups in city (miles)	*x*	0.5	1.5	2.5	3.5	4.5	5.07	5.07
Boundary of city development (miles)	*X*	0.5	1.5	2.5	3.5	4.5	5.07	5.07
Boundary between income groups in suburbs (miles)	*y*	5.07	5.07	5.07	5.07	5.07	5.5	6.5
Boundary of suburban development (miles)	*Y*	6.69	6.69	6.69	6.69	6.69	7.02	7.83
Poor household's utility change on moving from suburbs to city ($/yr)		-3,383	-3,155	-2,927	-2,698	-2,470	-537	-385
Rich household's utility change on moving from city to suburbs ($/yr)		0	0	0	0	0	-2,571	-3,085

Table 3: Simulation results for Case 2

At $B = 5.5$ and 6.5

When B is set to 5.5 and 6.5 miles, the equilibrium structure is Case 2.2. There are no longer rich households in the suburbs bidding to get into the city, and rents in the city fall: the rent decrease increases the utility achieved by rich households. Of course, as B increases in this range, poor households commute further and their utility decreases.

4. WELFARE COMPARISONS

4.1 Efficiency comparisons

Because we have specified an utility function with the property that the marginal utility of income is unity, the "as if" shifting of income from absentee landlords to resident households does not change the total surplus. Therefore our measure of efficiency is the sum of the average utility of resident households plus the average land rent.

The first-best efficient outcome is to separate the rich and poor households into different communities, so that there is perfect sorting of households by their taste for the public service (Tiebout sorting). Because commuting costs increase with income, rich households should be placed in the city and poor households should be placed in the suburbs. The city boundary should be set at 5.1 (miles) and there should be marginal undeveloped city land. The efficiency measure (average utility plus average land rent) for this equilibria is 23,915 ($ per household per year).

If the jurisdictional boundary is not 5.1 miles, should the city have a majority of rich households or a majority of poor households? If the city radius is small, the income class which lives in the city is also present in the suburbs where it obtains a non-optimal public service level. In contrast, all households of the income class which forms the majority in the suburbs live in the suburbs and obtain their preferred public service level. Relative efficiency involves a trade-off as to which income-class should be perfectly matched with its preferred public service, and considerations of commuting cost. We denote the public service voted by poor (rich) households as g_1 (g_2). Our specific utility function implies that matching is more important for rich households: the benefit to a rich household of being matched with g_2 instead of with g_1 exceeds the benefit to a poor household of being matched with g_1 instead of with g_2 ; or

Metropolitan population, *N = 270,000*							
Attributes of metropolitan area	Equilibrium values						
City jurisdictional boundary, *B* (miles from center)	0.5	1.5	2.5	3.5	4.5	5.5	6.5
<u>*CASE 1* (poor are majority in city)*:*</u>							
Efficiency measure (Average utility plus average rent) ($ per household per year)	23,459	23,456	23,469	23,507	23,548	23,357	23,182
City population of poor households	1,774	15,969	44,357	86,940	135,000	135,000	135,000
City population of rich households	0	0	0	0	6,443	31,726	52,207
<u>*CASE 2*: (rich are majority in city)</u>							
Efficiency Measure (Average utility plus average rent) ($ per household per year)	22,947	23,022	23,174	23,401	23,703	23,876	23,791
City population of poor households	0	0	0	0	0	0	0
City population of rich households	1,311	11,803	32,785	64,258	106,222	135,000	135,000

Table 4: Efficiency comparison of Case 1 and Case 2 equilibria

Table 4 presents the efficiency measures for the equilibria achieved under Cases 1 and 2, as well as the number of poor and rich households living in the city at each boundary size. If the city radius is small (3.5 miles or less), efficiency is higher when the poor are the majority in the city. If the city boundary is 0.5 miles, the efficiency difference is 512 ($ per households per year). We conclude that, for small cities, good matching of rich households with their desired public service is more important than considerations of commuting cost.

As the jurisdictional boundary expands, the number of households living in the city increases, fewer suburban households are being mismatched with their desired public service, and the importance of matching decreases. However, for Case 1, commuting costs increase as all (or almost all) rich households are being forced to live further out. In consequence, the welfare difference between the two cases falls. When the jurisdictional boundary is 4.5 miles, in the Case 2 equilibrium 80% of rich households are able to live in the city and obtain their desired public service; commuting cost considerations dominate and efficiency is higher under Case 2.

For jurisdictional boundaries of 5.5 and 6.5 miles, the city is able to contain all the households of an income class, and matching is not an issue. Minimization of commuting costs dominates, and higher efficiency is obtained when rich households form the majority in the city (Case 2). This is true even if there is undeveloped land.

To reinforce the importance of good sorting of rich households into the community providing their desired public service, we compare Case 1 outcomes with city radii of 3.5 and 6.5 miles respectively. The efficiency measure drops by 325 ($ per household per year). In the former case, 48,060 poor households live in the suburbs and get more public service than desired; in the latter case, approximately the same number of rich households (52,207) live in the city getting less public service than desired. Further analysis shows that approximately 2/3 of the drop in efficiency is associated with the perfect matching with public services changing from being with rich households to being with poor households, and approximately 1/3 of the drop is associated with the higher commuting costs.[18] [19]

4.2 Equity comparisons

Our measure of equity is the Rawlsian welfare function, *max min* $[U_1, U_2]$ where U_i is the utility achieved by a household with income M_i. Comparison of Tables 2 and 3 shows that the best outcome for poor households is for poor households to be the majority in the city (Case 2) and for the city boundary to be 5.5 miles. The utility of a poor household in this equilibria is 11,158 ($ per year) while the utility of a poor household in the first-best efficient outcome is only 10,843 ($ per year).

If the city radius is small (3.5 miles or less), Tables 2 and 3 show that poor households achieve utility 9,941 ($ per year) in the Case 1 equilibrium and 10,843 ($ per year) in the Case 2 equilibrium. Equity therefore places rich households in the city and makes poor households the majority in the suburbs. An important difference betwamoythe

If the city radius is large (4.5 miles or larger), Case 1 has all poor households living in the city and it is rich households who compete for city land. The low public service in the city makes living in the city relatively unattractive to rich households, creating a "barrier to entry" for rich households. This drives down the rent to benefit poor households. The benefit of good matching of poor households with their desired public service is no longer captured by landlords as rent. Case 1 is preferred for equity.

Our equity finding has a policy implication. Many U.S. metropolitan areas stylistically resemble Case 1.1 in which only poor households reside in the city and in which rich and poor households reside in the suburbs. In this case, expanding the city boundary to include the near suburbs (i.e. to shift the equilibrium from Case 1.1 to Case 1.2 or Case 1.3) is good policy for city residents as it increases their utility. It should be stressed that this utility gain accrues to poor families not because of tax shifting - all city households pay the same tax - but because of rent changes.

4.3 Conflict between efficiency and equity

The discussion in the two previous subsections has shown that efficiency and equity favor different city majorities. Table 5 summarizes the discussion.

Metropolitan population, *N = 270,000*

Attributes of metropolitan area	Equilibrium values

5. SUBURBANIZATION IN A GROWING CITY

Current metropolitan areas have grown out of much smaller cities. Does the likely path of development select an equilibrium which is efficient or which is equitable? Our presumption is that increases in the population are accommodated by marginal changes in the boundary between the income groups and not by large population shifts between the communities. Put differently, once an equilibrium configuration of majorities is established, we presume that it is maintained as the population grows (provided the configuration continues to be an equilibrium).

Historically, when metropolitan populations were small compared to the size of the city, all households lived in the city and poor households formed the majority. As the metropolitan population grows, rents increase in the city, the limit of urban development moves outwards, and the poor households who reside at the edge of development are pushed further from the city center. Eventually such growth will lead to the development of suburban communities. One possibility is that poor households are the first households to move into the suburbs. Poor households have no incentive to set up a suburban community while there is still undeveloped city land - if the household at the edge of the city were to move, he would get the same public service, pay the same rent and incur higher commuting costs. Unless rich households "jump over" undeveloped city land to form a new community in the suburbs, the suburbs are not developed until the limit of development moves across the jurisdictional boundary; at this instant poor households who reside at the edge of urban development spill into the suburbs. With poor households leaving the city, rich households become the city majority.[20]

The alternative possibility is that the rich households are the first households to move into the suburbs. By setting up a new community in the suburbs, they obtain their desired public service and face lower rents, but incur higher commuting costs. If they are sufficiently sensitive to the public service level, rich households choose to suburbanize while there is still undeveloped city land[21] corresponding to Case 1.3 in our simulations. In this scenario, rich households establish themselves in the suburbs, leaving poor households to control the city. Given our presumption that the equilibrium configuration of majorities does not change spontaneously (provided it continues to be an equilibrium), rich households control the city only if Case 1.3 does not arise while the metropolitan population is too small to completely fill the city.

To consider these possibilities, we approximate the path of development by the comparative statics of increasing the metropolitan population N in the presence of a fixed city boundary B=5.0 miles; other parameter values are maintained at the values shown in Table 1.[22] With a boundary at 5.0 miles, the city can contain all households until the population exceeds 150,811. At a very small population (not shown in the table), all households live in the city and the suburbs are uninhabited: the potential cost of commuting from the suburbs deters households from setting up a new community in the suburbs. Table 6 shows the form of the different equilibria as N is increased in increments of 100,000 households. At a metropolitan population of 100,000 households, there is an equilibrium of the form of Case 1.3: although the metropolitan population is still too small to fill the city, rich households "jump over" undeveloped city land to form a new community in the suburbs. Our earlier discussion, therefore, suggests that rich households become established in the suburbs, or that the path of development selects the Case 1 equilibrium - poor households form the city's majority.

City jurisdictional boundary: *B = 5.0* (miles)

	Equilibrium outcomes
Metropolitan population, *N*	

Table 6 shows that, at *N = 100, 000* households, two equilibria exist: Case 1.3 (poor households are the city majority), and Case 2.2 (rich households are the city majority). We consider Case 2.2 to be unlikely. Its realization presupposes that the path of development allows rich households to become established as the city's majority, or that poor households migrate to the suburbs. However, as discussed above, the incentives do not favor such a migration - while poor households form the city majority, migrating to the suburbs makes a poor household worse-off.

As the metropolitan population further expands from 100,000 to 800,000 households, the possible equilibria with the poor forming the city's majority moves from Case 1.3 to Case 1.2 to

6. CONCLUSION

This paper has examined a monocentric urban model in which the metropolitan area is divided into two jurisdictions -a central city and a surrounding suburb - and there are two income classes. There are multiple equilibria: those in which poor households are the city's majority and those in which rich households are the city's majority. We find that there is often a conflict as to which equilibrium is preferred. For small central cities or large metropolitan population sizes, efficiency favors the poor being the city's majority and equity favors the rich being the city's majority. For large central cities or small metropolitan population sizes, the ordering is reversed. This conflict arises from the basic spatial allocation problem and the competition for land. Because of the importance of matching rich households with their preferred public service, efficiency favors an equilibrium in which rich households are the majority in a community that is large enough to contain all rich households although this may involve higher commuting costs. Alternatively, equity favors an outcome in which the poor are a majority in a community that can contain all poor households. Otherwise, poor households that reside in the rich community will bid up the price for land in the poor community, and landlords will capture all benefits from the matching of poor households to their desired level of public services. Although the model is necessarily stylized, we believe it highlights important trade-offs in urban policy and in the growth of metropolitan areas.

If $y = Y$

Conversely, if the [redacted]

in the suburbs as [redacted]

$\pi (Y^2 -$ [redacted]

In addition we require that consumption is non-negative

$$x < X: \qquad M \qquad ;$$

APPENDIX : POSSIBLE EQUILIBRIUM CONFIGURATIONS

With a majority of poor households, it is impossible for there to be a majority of rich households in both the city and in the suburbs. The three possible cases are represented in the figure below:

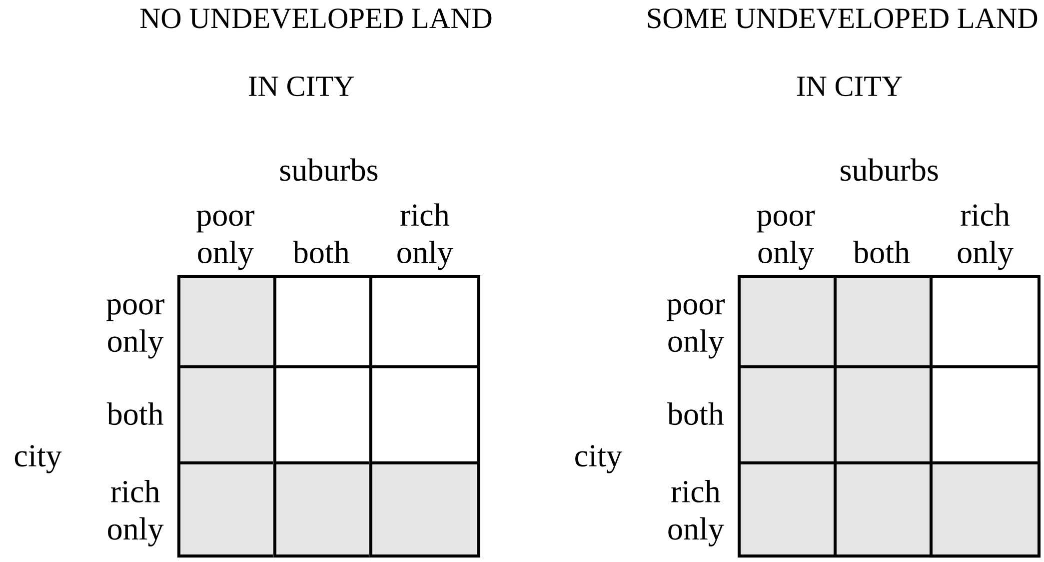
		SUBURBS MAJORITY	
		poor	*rich*
URBAN AREA MAJORITY	*poor*	*Case 3*	*Case 1*
	rich	*Case 2*	

Figure : classes of equilibria

These cases are further broken down into possible sub-cases in the matrix of matrices shown below.

CASE 1:

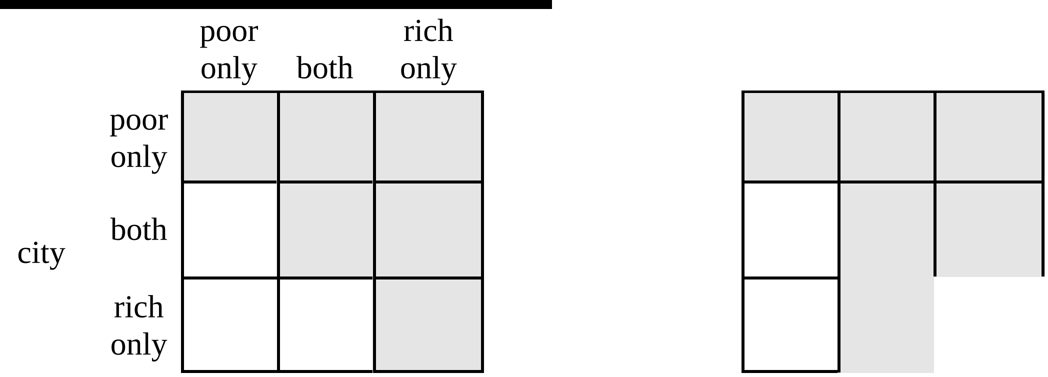
poor majority in city, rich majority in suburbs



[redacted]

CASE 2:

rich majority in city, poor majority in suburbs



[redacted]

The outer matrix is to be interpreted as follows. The first column corresponds to the cases in which there is no undeveloped city land; the second column corresponds to the cases in which there is some undeveloped city land. The first row corresponds to Case 1 (poor households form the majority in the city and rich households form the majority in the suburbs); the second row corresponds to Case 2 (rich households form the majority in the city and poor households form the majority in the suburbs); and the third row corresponds to Case 3 (poor households form the majority in the city and in the suburbs).

Within each cell of the outer matrix is a matrix. The first column of this submatrix corresponds to the case when the suburbs contains only poor households; the second column corresponds to the case when the suburbs contains both poor and rich households; and the third column corresponds to the case when the suburbs contains only rich households. Similarly, the first row corresponds to the case when the city area contains only poor households; the second row corresponds to the case when the city contains both poor and rich households; and the third row corresponds to the case when the city contains only rich households;

The boxes are shaded if they do not represent possible equilibria. They are coded with a letter which gives the cause as to why the configuration is inconsistent with an equilibrium:

A: The composition is inconsistent with the metropolitan population. I.e., it is not possible for city and suburbs to both contain only poor households or only rich households.

B: The composition is inconsistent with assumed voting majorities. E.g., Case 1 (poor households form majority in both areas) is inconsistent with the city having only rich households.

C: If rich households vote the public service in the city, they are willing to pay more than poor households for both the commuting advantage of the city and for the public service of the city. Therefore rich households outbid poor households for locations in the city, so that either the city contains only rich households or the suburbs contain only poor households. (Proposition A below).

D: If a group forms the majority in the city, any member of that group that resides in the suburbs could raise their utility by moving to undeveloped land in the city. This move would provide the suburban member with lower community costs, a no-worse fit in terms of public service provision, and no increase in housing price since the undeveloped land must be priced at the reservation value of land. Hence, if there is undeveloped land in the city, poor (rich) households cannot form the majority in the city and live in the suburbs. (Proposition B below).

E: If poor households are the majority in both jurisdictions, they vote the same public service level in each jurisdiction. Rich households have a higher willingness to pay for the commuting advantages of the city and, with the same public service in each jurisdiction, outbid poor households for city homes. Hence, if poor households are present in the city, there can be no rich households in the suburbs. (Proposition C below).

The unshaded boxes correspond to the possible equilibria. The number inside of these boxes labels the case.

The propositions are shown overleaf. Denote the city's public service as g_c and the suburban public service as g_s.

If no rich households live in the city, $x=0$ and the above expression is the utility a rich household achieves if it moves to the city. A rich household living at

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ENDNOTES

1. Ross and Yinger (1999) survey this literature.

2. Inefficiencies include the impact of the property tax on housing consumption and the influence of community heterogeneity on voting outcomes.

3. Wheaton (1976) and Sasaki (1990) provide a comparative static analysis of this equilibrium.

4. The assumption of fixed housing size greatly simplifies the problem and allows us to avoid well-known existence problems associated with stratified local public-finance equilibria (see Epple, Filimon and Romer (1984, 1993)).

5. For ease of presentation, the jurisdiction is assumed to provide a public service and not a public good. The public service shows constant returns to community size. It is straightforward to change the publically-provided good from a public service to a public good.

6. If *dr/dd* is interpreted as the slope of the bid-rent curve of the household, Inequality (1) implies that rich households have steeper bid-rent curves then poor households, or rich households outbid poor households for the locations closer to the metropolitan center.

7. This assumption is consistent with the findings of Glaeser, Kahn, and Rappaport (1999).

8. A formal proof is provided in de Bartolome and Ross (2000).

9. Because of the specific form of the utility function, it is straightforward to change this assumption to allow each household to receive an equal share of the total rent paid as a lump-sum transfer. To do this, simply sum the average rent and the quoted utility levels in Tables 2 and 3. We prefer to have rents paid to absentee landlords as the alternative suggests that households in one community benefit from property value increases in the other community.

10. A list of all such possible equilibria is available from the authors on request. de Bartolome and Ross (2000) show that an equilibrium with strict income sorting always exists.

11. In many studies, D is required to be positive, which implies a price elasticity that exceeds unity. We generalize the utility function to allow for negative values of D and price elasticities that are less than one.

12.   Sensitivity analysis was performed using a lower and higher value (0.1 and 0.5) for the implied income elasticity of land demand, and there was little effect on the results.

13. This figure is a crude average and ignores possible variations in the number of households living in low- and high- service communities.

14. Societal figure calculated as local government expenditure financed from local government own-revenue (*Census of Governments Volume 4 Number 5* 1992, Table 3) divided by personal

income times one minus the average federal income tax rate (*Individual Income Tax Returns* 1990, Table 1.1).

15. Using the labels described later in the text, the equilibrium at $B = 4.5$ miles in Table 2 changed from Case 1.2 to Case 1.3.

16. At each value of B, we found only one equilibrium within each Case. Because the two income classes have been chosen to be the same size, we found no equilibrium in which one income-class forms the majority in both communities. We also did not find an equilibrium in which both communities contain both income classes.

17. This is the average rent paid per household. If the value quoted in the table is T ($ per household per year): because the average lot size is 0.3333 (acres per household), the average rent per acre per year is $3T$. Because the reservation rent has been set to zero, this is interpreted as the average rent premium (above the reservation rent) paid for one acre of land in the metropolitan area.

18. Consumer surplus from public services falls by 216 ($ per household per year). Commuting costs increase by 109 ($ per household per year).

19. We have compared above the efficiency of the two equilibria achieved at a given city boundary. For completeness, we now consider the effect of an increase in the city's jurisdictional boundary with consideration initially focused on Case 1 (city majority of poor households). As the boundary moves from 0.5 to 4.5 miles, efficiency does not change very much: the improvement of the matching of poor households with public services is almost exactly balanced by the increase in the commuting costs because the rich are being pushed further out. Initially (as the boundary moves from 0.5 to 1.5 miles), the increase in commuting costs dominate and efficiency falls. However, as the city becomes larger, the geometry of the problem means that, for each mile the boundary moves out, the area of the city expands by a larger amount and the average distance of rich households from the city center increases by a smaller amount. In consequence, there is a larger gain in the matching of poor households with their desired public services and a smaller increase in the commuting cost of rich households. Between 1.5 and 4.5 miles, the improvement in matching dominates and efficiency increases slightly.

Continuing with Case 1, as the boundary increases from 4.5 to 6.5 miles, some rich households are migrating into the city, lowering their commuting costs; however, the rich

20. With equal numbers of poor and rich households, the rich become the majority in the city immediately the poor spill over into the suburbs.

21. Although the causation is quite different, this result resembles the leapfrog development pattern that may appear in models of urban growth where some land is left vacant in the interior because its option value for future development exceeds its value in current use. For some examples in the literature, see Arnott and Lewis (1979), Capozza and Helsley (1989) and Wheaton (1982).

22. Technically, to ensure that there is a metropolitan majority of poor households, we assume that there is one more poor household than rich household.

23. If households of income M_i do not live in the community j, we interpret below c_{ij} to be the rent plus commuting cost which a household of income M_i would pay if he were to move into the community j.