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Technology Life-Cycles and Endogenous Growth

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Abstract

1. Introduction

Besides the fact that rapid and sustained technological change is a relatively recent phenomenon, the history of technological progress reveals three distinct empirical regularities. First, there exists a strong complementarity between inventions and innovations in expanding the technology frontier. Without inventions, the innovative process will eventually be subject to decreasing returns, and absent productivity-enhancing innovations, new technologies may never be adopted. Second, major breakthroughs in technology arrive infrequently and in clusters. It is well documented, for example, that the height of the ancient Greek civilization between 400 B.C. and 100 A.D., the Ming dynasty rule in China during the 14th century, and more recently, the Industrial Revolution of the 18th century were periods during which many new and path-breaking discoveries were made. Finally, every new technology appears to go through a three-period life-cycle; the during which the newly discovered technology is adopted and the potential bene¯ts of learning-by-doing are largest, the macro- and microinventions (or inventions versus innovation no

here, they do not incorporate learning-by-doing in

$$= (1 - \quad) \left(\frac{\quad}{\quad} \right) . \tag{2}$$

2.2. Individuals

Individuals live for two periods in overlapping generations. They are endowed with one unit of time in every period. In both periods, individuals supply their labor inelastically; in the first period of life, they work and save, and in the second period, they work, dissave and consume. Individuals' preferences are represented by a utility function that is linear in consumption in the second period.[6] There is no population growth.

2.3. The Technology and Potential versus Actual Productivity

Firms must purchase new machines in every period because machines depreciate fully in one period. Let and respectively denote the quality and quantity of machines utilized in production at time and let represent the underlying level of technological sophistication in the same period.[7] I assume that the machinery aggregate used in production at time , is given by the following:

$$= \frac{\quad}{\bar{\alpha}} \tag{3}$$

Equation (3) implies that the machinery aggregate, , increases with the number and quality of machines used in production as well as with their underlying level of technology.

Based on this specification, technological progress can come about in two ways: inventions and innovations. The former is the discovery of new technologies (or leaps

[6] This assumption pins down the interest rate at the discount rate. Neither relaxing this assumption nor allowing consumption in the first period would materially affect the main results.

[7] One can think of the term Á z as the overall quality of a given machine. With this formulation, I am essentially adopting the notion that changes in each machine's overall quality can be decomposed into two components, one of which is conditional on the level of the existing technology and a residual which is directly determined by the technology itself.

up the quality ladder a la Grossman-Helpman and Aghion-Howitt where i

¡ = . Thus, the underlying productivity of the machines in use, , continues to improve according to $_{+1}$ = exp[1 (+ 1)(+ 2)] = ^{1}exp[¡ 1 (+ 2)] and economic growth remains positive. Over time, however, growth monotonically decreases and asymptotically converges to zero as the exogenously given level of technology constrains and exhausts the potential ntn sal ohaut ttttbe0tesv TTDD 0 0. 1 13 3 2 2 TTc c ((a a)) output (which will be a function of ¹) will be sustained and the economy will follow a Balanced Growth Path (BGP) in the long run.

2.4. Adoption of New Technologies and Next Generation Machines

The decision of a ¯rm 2 [0 1] is

$$\max_{\substack{j \\ t} \; \substack{j \\ t} \; \substack{j \\ t} \; \substack{j \\ t}} \quad (\quad) \; (\quad)^{1 ¡} \quad ¡ \qquad ¡ \tag{8}$$

where is given by (6) and denotes the price per machine which the ¯rm takes as given. The solution to this problem yields, 8 2 [0 1]

$$= \quad ^{1} \exp \; ¡ \; \frac{1}{1 + \quad ¡} \quad \frac{(\quad)^{®} (\quad)^{1 ¡}}{} \quad ^{\frac{}{¡ ®}} \tag{9}$$

As (9) implies, the demand for machines is strictly increasing in their vintage, ¡ , and the number of times machines with that underlying technology has been improved,

Lemma 1: $\frac{\substack{j \\ t}}{(\; ¡ \;)}$ 0 and $\frac{\substack{j \\ t}}{t}$ 0

Proof:

$$\frac{}{} = \frac{1}{1 \; ¡} \frac{}{(1 + \quad ¡ \quad)^{2}} \qquad 0 \tag{10}$$

and,

$$\frac{}{(\; ¡ \;)} = \frac{}{1 \; ¡} \frac{}{2} \qquad 0 \tag{11}$$

$$= \min \Big\{ \frac{}{} \quad ^{-1} \exp \left(- \frac{(\; - 1)}{} - \frac{}{1 + \; - } \right) , \Big\} \qquad (13)$$

Proof: See Appendix, Section 6.1.

Note that, the monopoly price of both newly invented machines and newer generation of vintage technology machines are non-increasing in the number of times a given technology machine has been improved via innovations. The reason for this is that, existing older machines, which are available at marginal cost become more productive with the introduction of each new generation machine. Consequently, regardless of whether newest machines embed a totally new invention or they belong to a newer generation of existing machines, the monopolist's price declines as existing, alternative, machines become more productive. Note also that the monopoly price of machines which embed a newly discovered technology is non-increasing in the length of time the previously-superior older technology has remained in production (i.e. $\; - \;$). This is due to the fact that learning-by-doing improves the productivity of technologies which have stayed in use longer (that is, regardless of how many different generation of machines with a common underlying level of technology have been introduced over a technology's tenure).

Lemma 3: $\forall \; \geq \cdot \leq [(1 - 4(\; \ln \;))^{-} - 1] \; 2,$

$$\frac{}{(\; - \;)} = 0 \qquad \frac{}{} \quad 0;$$

$$\forall \; \geq \wedge \; \leq \{ \; [(\; - \;) \; (1 + \; - \;) - \ln \; - \; (\ln ^{1} - 1)] \},$$

$$\frac{}{(\; - \;)} \quad 0 \qquad \frac{}{} \quad 0 \qquad \frac{}{_1} \quad 0$$

Proof: (i) 8 ¸ · = exp ³ ———

Lemma 4: 8 ¸ ·,

—— ;

8 ¸ ^ ,

—— 0

Proof: (i) = (¡) and 8 ¸ ·

$$—— = ¡ \frac{———}{1\ ¡} — ^{3} \quad ´ \quad ¡ — \quad 0 \tag{16}$$

(ii) = (¡) and 8 ¸ ^

$$—— = ¡ \frac{———}{1\ ¡} — ^{3} \quad ´ \quad ¡ — \quad 0 \tag{17}$$

¤

2.5. Equilibrium R&D E®ort in Inventions versus Innovations

Inventions and innovations are the result of R&D carried out by research ¯rms which use the ¯nal consumption good as their only input. In all time periods, there are a ¯nite number of exogenously given R&D ¯rms, , who behave competitively.[10] Let denote the economy-wide probability that a new invention will actually occur in any given period and denote the economy-wide probability that a next generation machine will be introduced in . I assume that these probabilities, and , depend positively on aggregate resources spent on R&D in inventions and innovations, respectively:

$$¤ = {}^{1} \left(\frac{¤}{1 + ¤} \right) ; \quad ¤ = \quad ; \quad 0 \quad {}^{1} \cdot 1; \quad 0 \quad 1 \tag{18}$$

[10] 8)8 ; 8

where ¤ is the aggregate resources spent on R&D in period for the purpose of innovating or inventing, and where 1, 0 1 · 1 denotes the maximum attainable economy-wide probability of invention or innovation in any given period .

If aggregate inventive or innovative activity is successful in advancing the economy-wide level of technology in use, the probability that any given R&D firm lands the monopoly rights to sell new technology machines (or those which belong to the next generation of an existing technology) depends on the relative share of resources the firm spends on R&D, ¤ ¤ (= 1 2 3). Put differently, conditional on the fact that an invention or an innovation has occurred in any given period , the odds of a particular R&D firm being the inventor or the innovator of that new technology depends positively on the ratio of its R&D expenditures to that in aggregate for that R&D activity.

Monopolists' patents expire after one period. Thus, if the technology does not become obsolete after one period, consumption-goods firms can replace existing machines at their marginal cost, . (Because the machines depreciate fully in one period, producers must purchase new machines in every period.) The decision of an R&D firm, , = 1 2 3 is

$$\max_{A_t;n,\, x_t;n} \left\{ (1 - \quad) \frac{\quad}{\quad} + (1 - \quad) \frac{\quad}{\quad} - (\quad + \quad) \right\} \quad (19)$$

where (1 −) ( ) and (1 −) ( ) respectively denote the expected monopoly profits from inventions and innovations, 0 is the marginal cost of the R&D effort in terms of the consumption good[11], and where ¤ = (¤ −) ¤ = $\int_0^1$ ¤ . Implicit in (19) is the notion that when a new invention and an innovation are made simultaneously, the monopolistic competition in setting prices generates a Cournot-Nash equilibrium outcome in which =.5 0 Tc (t) Tj 4.5 0 TD 0.132 Tc (a) Tj 6 0-5 0 TD 0.85

payo® from an invention when there is also an innovation, and the expected monopoly payo® from an innovation when there is also an invention, , are both zero.

Proposition 1:

8 = 1 2 3

$$\sim \quad = \frac{(1 \text{ ¡ } \quad)}{} \qquad \text{and} \qquad \sim \quad = \frac{(1 \text{ ¡ } \quad)}{} \tag{20}$$

~ ~ = ^{1}[~ (1 + ~)]

= ^{1}[~ (1 + ~)]

Proof: See Appendix, Section 6.2.

¤

Not surprisingly, aggregate equilibrium R&D e®ort in inventive or innovative activity, ~¤ ~¤ = ~¤ , ¤ = is increasing in monopoly pro¯ts from that invention or innovation.[12] [13]

[12] By assumption, there is free-entry into research and development by relatively small ¯rms. Those ¯rms ignore their impact on both the economy-wide probability of success in generating new inventions and the total number of R&D ¯rms (which in turn a®ect the conditional odds of landing monopoly rights). If there had been one large ¯rm engaged in R&D, it would have taken into account the e®ect of changes in its R&D resources, !¤, on the probability of invention, I , but the qualitative nature of the results would have been una®ected. Similarly if there had been barriers to entry into the R&D sector which would have restricted the number of ¯rms engaged in research and development, I would have had to consider a game-theoretic solution but again the qualitative nature of the main results would have remained intact.

[13] As also implied by (20), the intensi¯cation of research and development activity might be related to more ¯rms deciding to invest in R&D. This result would be consistent with Sokolo® and Kahn (1990) who discuss the historical pattern of entrepreneurial activity which eventually led to inventions. Late 18th and early 19th century patent data indicate that it was the broadening of the entrepreneurial pool, rather than the concentration of inventions in the hands of a limited group of researchers and professional inventors, that led to rapid technological change in the United States in the 19th century.

Proposition 2: 8 = 1 2 3

$$\frac{¤}{¤} - ¤ = \tilde{}¤ \quad 0 \quad ¤ =$$

· ¸ 2. More explicitly, the second term in (A.1) guarantees that the t

e±ciency gains from learning-by-doing with that technology have been realized. Second, since R&D ¯rms have updated the quality of machines numerous times, innovations have also led to greater e±ciency in the use of the underlying level of technology, . Thus, in this maturity phase, the productivity of machines in use is higher than that of machines in the two other regimes, and the marginal productivity gains that would result from the adoption of a newer-generation machine or a newly-invented technology is relativelyaon t esult f d i 0

i + 1 · i 0

[Table 1 about here.]

In both simulations, most parameter values such as the asymptotic technology and machine quality levels, ^{1} and ^{1}, marginal cost of machine production, are arbitrary, and are set at their chosen values for convenience. The initial level of the technology, $_0$, is deliberately set very close to zero. In the first simulation, the asymptotic probabilities of invention and innovation, ^{1} and ^{1}, are set at 10 percent and 90 percent respectively. That is, when resources devoted to R&D for invention and innovation are infinite, the economy-wide odds of a commercially successful invention are 10 percent and that of innovation are 90 percent. The values of the quality jumps, ^{1}, and the parameter are chosen such that (A.1) is satisfied. Specifically, ln ^{1} = 2 3 (1 +) = 2 14 and ¡ ln = 0 145 1 6. As a result of the latter, a monopolist who holds a patent for a newly innovated machine of generation 3 or less can charge the unconstrained monopoly markup . And given that ln ^{1} = 2 3 in the first simulation the invention of a new technology leads to an approximately tenfold increase in the parameter holding constant machine quality, , and vintage, ¡ It is important to note, however, that this does not imply a tenfold increase in machine productivity immediately after the introduction of a new technology. For example, due to the e®ects of learning-by-doing and machine vintage o®er ®a nmidyn0 tt one which lasts from 500 B.C. to 300 A.D. when there are 7 new discoveries, and ¯nalli4 v eTcs (0gti) gTajn4

6. Appendix

2

properties of ¤ ¤ = suggests that ¡ 1 h (1 ¡ x t)

h

B ¼ 1 1) t9 1 ¡

)108Tj 5.2522.25 .31 0.11 Tf 00.132 Tc (s) Tj 5.25+0 TD31F0 12 Tf 0 -0.0533 Tc (l) Tj 4.5 3 TD /F12 0.11 Tf 0.318 Tc (x054) Tj -0.75 -8.25

Finally, (6.6) and Lemmas 1-4, 8 + 1 · · and ¡ 0,

$$\frac{(\quad)}{(\ \text{¡}\ )} \quad 0 \quad \text{and} \quad \frac{(\quad)}{} \quad 0 \qquad (6.8)$$

which implies that, 8 + 1 · and ¡ 0, $\frac{(\ \acute{A}_t \ \ x_t)}{(\ \text{¡}\ )}$ 0 and $\frac{(\ \acute{A}_t \ \ x_t)}{t}$ ¸ 0

□

Matsuyama, K. (1999). \Growing Through Cycles," , 67 (2), March, 335-347.

Mokyr, J. (1990). , (New York: Oxford University Press).

Romer, P. M. (1990). \Endogenous Technological Change," 98 (5), October, 571-602.

Schumpeter, J. A. (1934). (London: Geo®rey Lumberlege, Oxford University Press).

Sokolo®, K. L. and B. Z. Khan. (1990). \The Democratization of Invention During Early In
l50(a)TDj 6.234 TDc-0.06 TT6.05 0Tb2a 0.162 Tc (f) Tj 3.64 Tc (I) Tj 4.5 0 TD 0.234
0 T6j 6.75 Tc (h) Tj 6.75 Tc (D) Tj 9 0 TD 0.05 Tc (e) Tj 5.2(E) (f) Tj 8.25 0 TD 0.TD 0.132 Tcr 0.2 TD

Figure 1

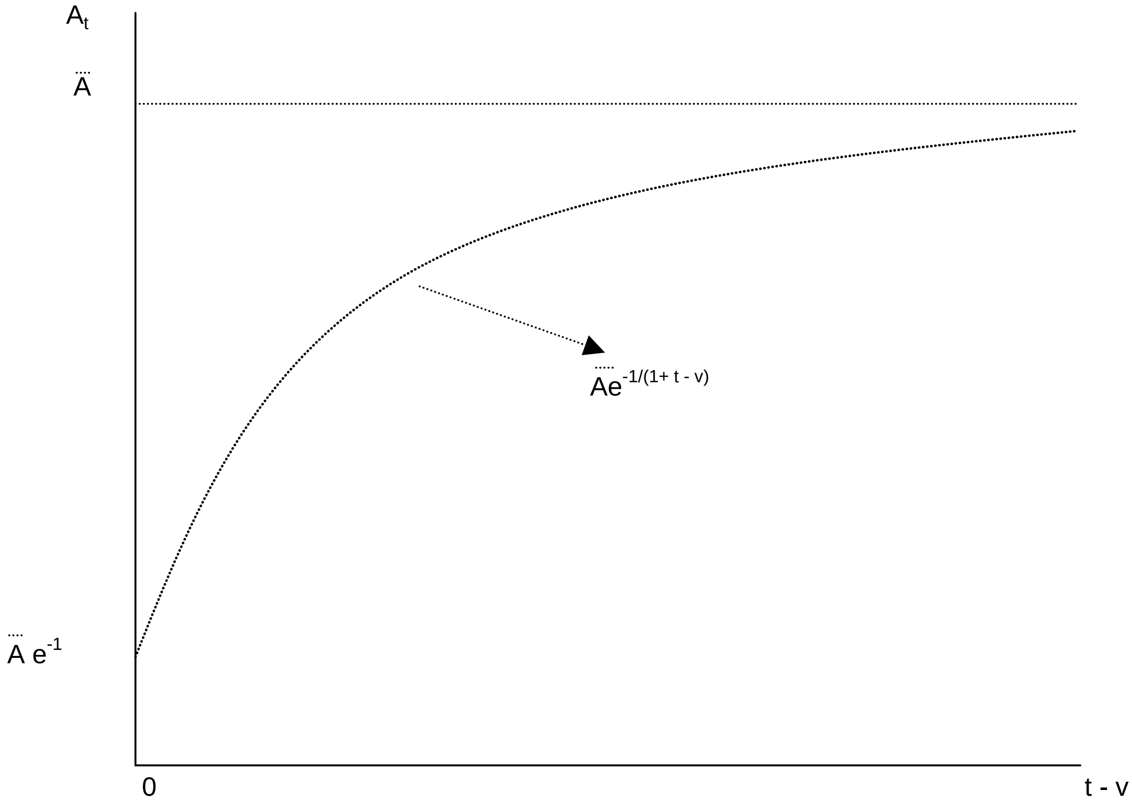


Figure 2

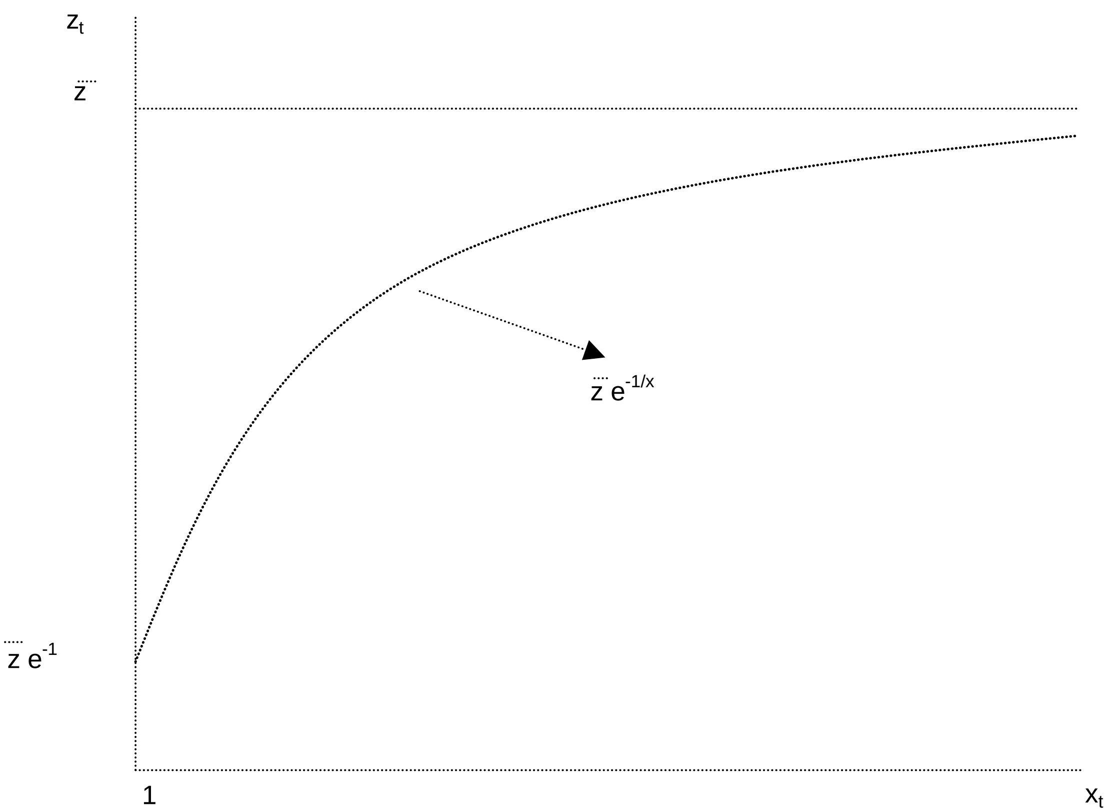


Figure 3

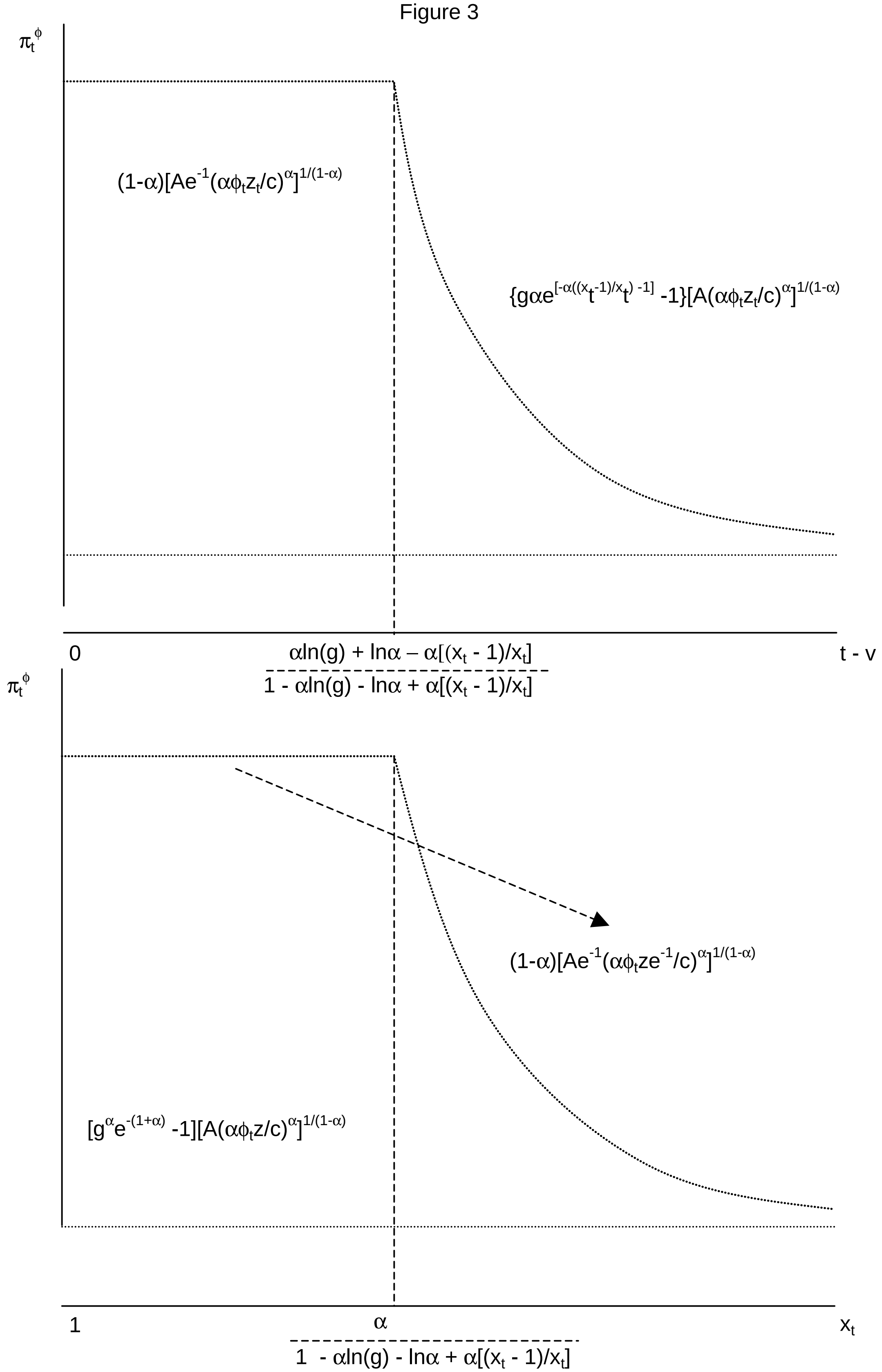
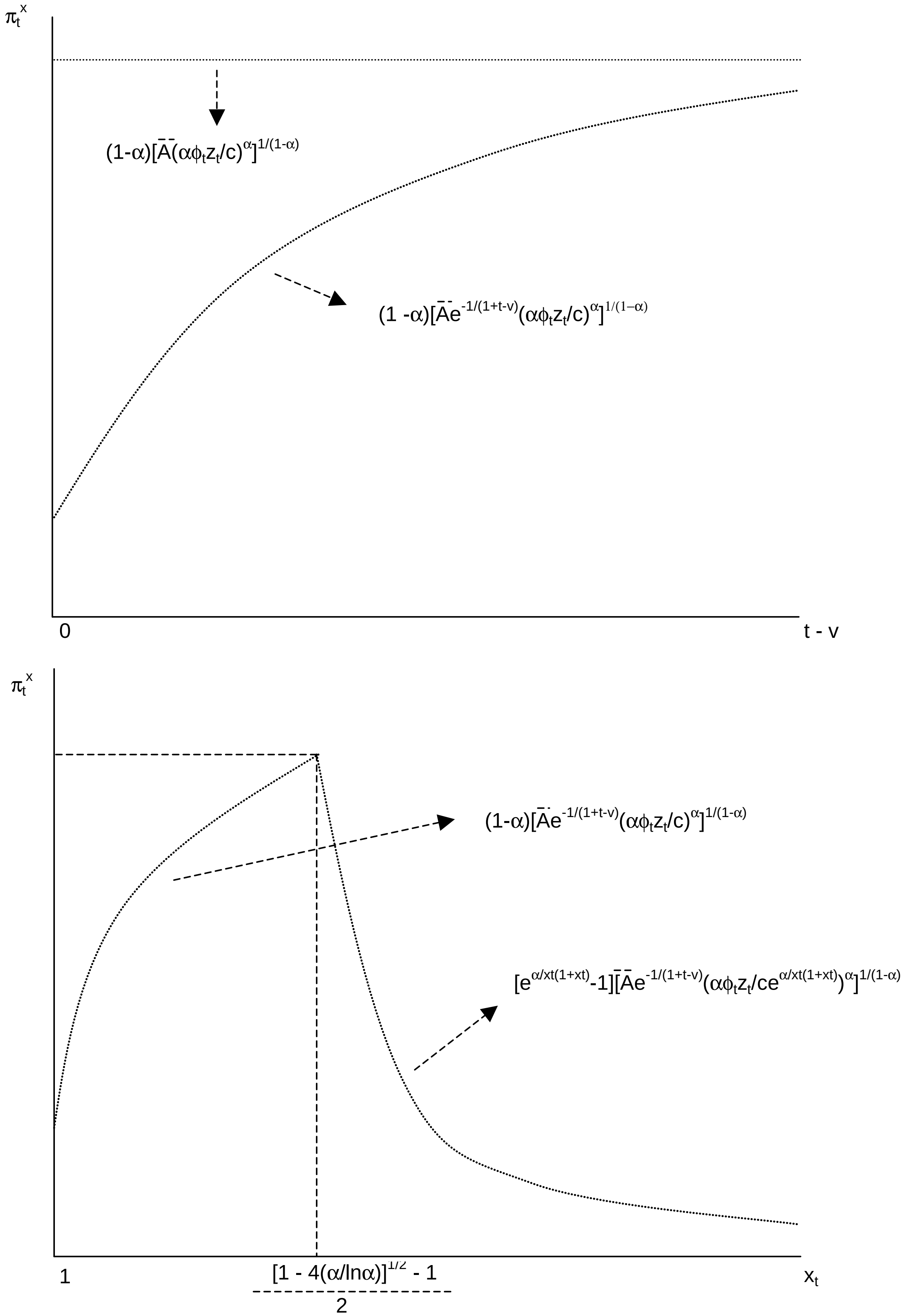

π_t^{ϕ}
$(1-\alpha)[Ae^{-1}(\alpha\phi_t z_t/c)^{\alpha}]^{1/(1-\alpha)}$
$\{g\alpha e^{[-\alpha((x_t-1)/x_t)-1]} -1\}[A(\alpha\phi_t z_t/c)^{\alpha}]^{1/(1-\alpha)}$
0
$\alpha\ln(g) + \ln\alpha - \alpha[(x_t - 1)/x_t]$
$1 - \alpha\ln(g) - \ln\alpha + \alpha[(x_t - 1)/x_t]$
t - v
π_t^{ϕ}
$(1-\alpha)[Ae^{-1}(\alpha\phi_t ze^{-1}/c)^{\alpha}]^{1/(1-\alpha)}$
$[g^{\alpha}e^{-(1+\alpha)} -1][A(\alpha\phi_t z/c)^{\alpha}]^{1/(1-\alpha)}$
1
α
$1 - \alpha\ln(g) - \ln\alpha + \alpha[(x_t - 1)/x_t]$
x_t

Figure 4



π_t^x
$(1-\alpha)[\bar{A}(\alpha\phi_t z_t/c)^{\alpha}]^{1/(1-\alpha)}$
$(1-\alpha)[\bar{A}e^{-1/(1+t-v)}(\alpha\phi_t z_t/c)^{\alpha}]^{1/(1-\alpha)}$
0
t - v
π_t^x
$(1-\alpha)[\bar{A}e^{-1/(1+t-v)}(\alpha\phi_t z_t/c)^{\alpha}]^{1/(1-\alpha)}$
$[e^{\alpha/xt(1+xt)}-1][\bar{A}e^{-1/(1+t-v)}(\alpha\phi_t z_t/ce^{\alpha/xt(1+xt)})^{\alpha}]^{1/(1-\alpha)}$
1
$\frac{[1-4(\alpha/\ln\alpha)]^{1/2}-1}{2}$
x_t

Figure 5:

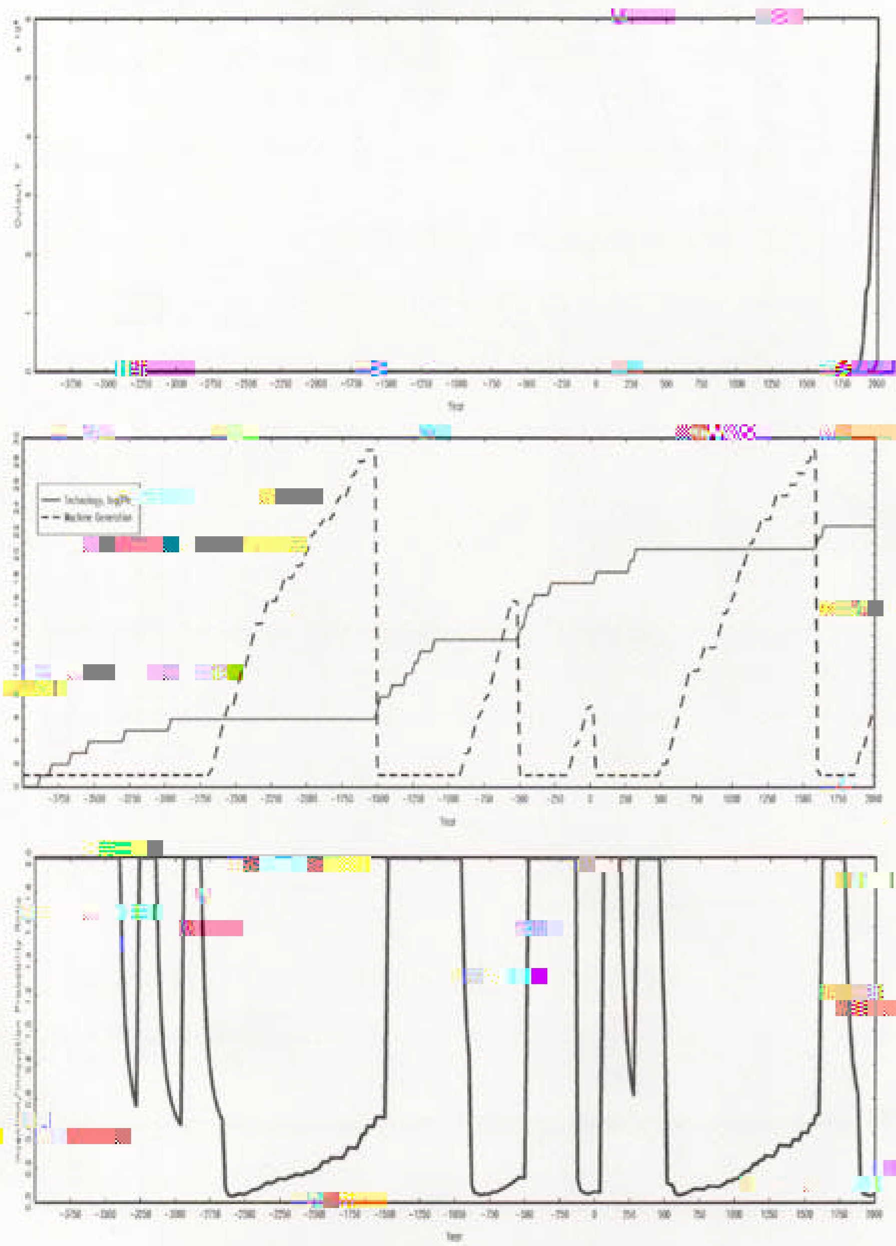


Figure 6:

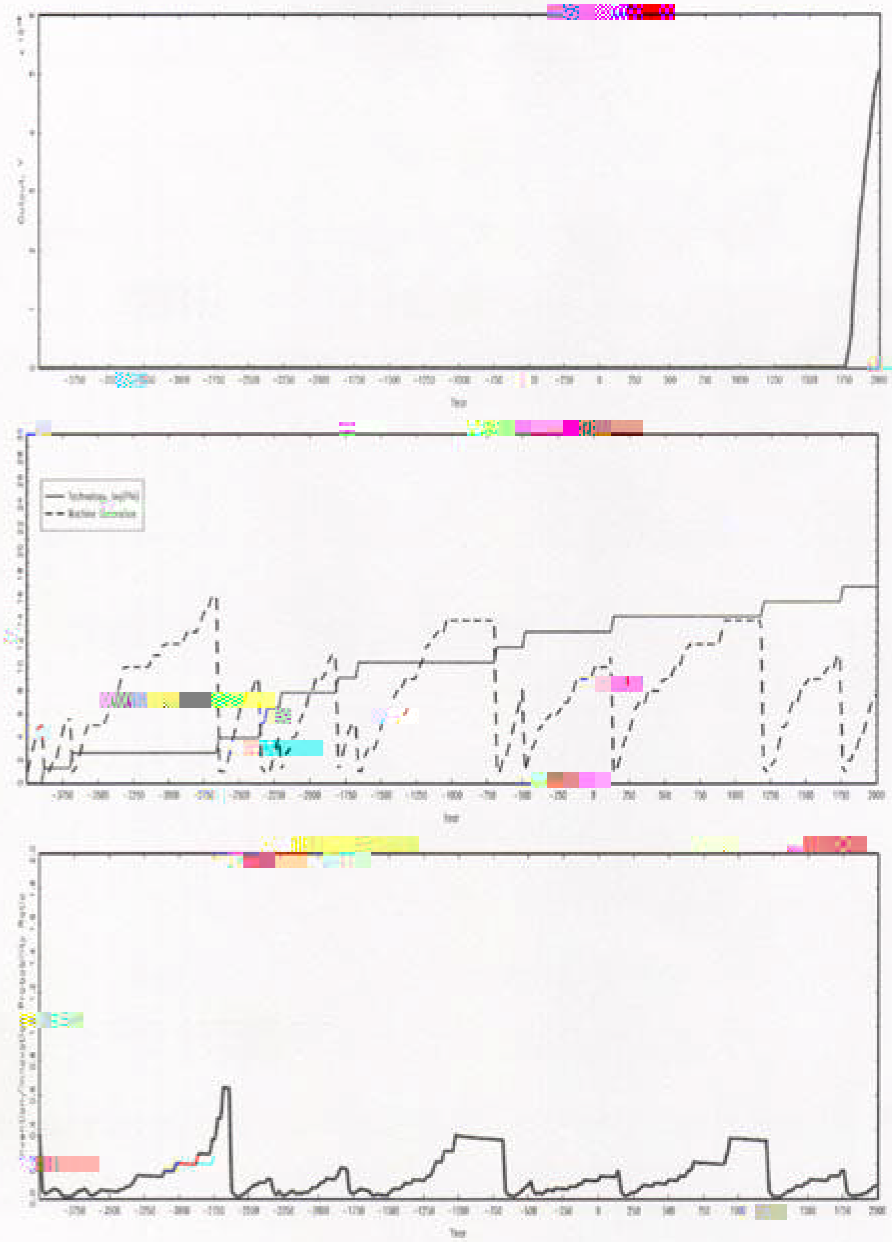

Output, Y
Year
Technology, log(PN)
Machine Generation
Year
Year

Table 1: Parameter Choices

Parameter	Value (I)	Value (II)
^{1}	1	1
$_{1}$	1	1
^{1}	0 10	0 05
^{1}	0 90	0 75
	10	10
	10	10
$_{0}$	$10^{i\ 20}$	$10^{i\ 20}$
$_{1}$	10	20
	0 875	0 675
	0 10	0 10