DISCUSSION PAPERS IN ECONOMICS

Working Paper No. 00-01

Does Evolution Solve the Hold-up Problem?

Tore Ellingsen
Department of Economics, Stockholm School of Economics
Stockholm, Sweden

Jack Robles
Department of Economics, University of Colorado at Boulder
Boulder, Colorado

January 2000

Center for Economic Analysis
Department of Economics

University of Colorado at Boulder
Boulder, Colorado 80309

Does Evolution Solve the Hold{up Problem?

1 Introduction

[illegible]

2 Investment and Bargaining

[illegible]

[illegible] $D_A := \{V \; I() - x; V \; I()\}$ [illegible]

[illegible] B [illegible] A [illegible] A [illegible] D [illegible] B [illegible] A [illegible]

[illegible] $I; y \; x()$ [illegible] A [illegible] y, $D \to D$ [illegible]

[illegible] A [illegible] $I := I^*$ [illegible] I

3 Evolution

population ... N ... $t \in \{1, 2, \ldots\}$ ... A ... B ... *beliefs* ... A ... B ... B ... A ... x

Assumption 1 *(i) The pie division is small:* ... *(ii) The population is large:* ...

adaptation

random mutation.

Θ

basin of attraction

μ *B* μ μ′

μ′ μ μ′ *single mutation neighborhood* μ μ′ ∈ *M* μ μ′ μ

X *mutation connected set*

μ_1 $\mu_n \subset X$ $\mu_2, \ldots, \mu_{n-1}$

k ; ...; *n*; *M* μ ∩ *BX*μ

[illegible]

$$x^L = \max\{x \in D_B(I^*) \mid (V(I^*) - x)\frac{N-1}{N} - I^* \geq V(I) - I\}.$$

[illegible]

$$x^M = \max\{x \in D_B(I^*) \mid V(I^*) - x - I^* > V(I) - I\}.$$

[illegible] $x^M \leq x^L \leq x^M$. [illegible]

$$\begin{aligned} V^* - x^M (N-1)/N - I^* &= V(I^*) - x^M - I^* - (V(I^*) - x^M)/N \\ &\quad - (N-1)/N \\ &> V(I^*) - x^M - I^* - x^M/N \\ &\geq V(I^*) - x^M - I^* \\ &\geq V(I) - I, \end{aligned}$$

[illegible] $(N-1)/N$ [illegible] x^L [illegible] B [illegible] A [illegible]

Proposition 3 *Let agents bargain according to the Nash demand game. The outcome ‰ is locally stable if and only if ‰ = {I*; V(I*) − x; x}, where* $x \leq x^L$.

[illegible]

[illegible] Θ [illegible] $r(\mu; \mu')$ [illegible] μ [illegible] μ'. [illegible] $\mathcal{G}$ [illegible] *resistance* [illegible] *cost* [illegible] $\mu \to \mu'$ [illegible] $r(\mu; \mu')$ [illegible] μ-*tree*, Γ [illegible] $\mathcal{G}$ [illegible] $\mu' \neq \mu$ [illegible] μ [illegible] Γ [illegible] *stochastic potential* [illegible] μ [illegible]

Theorem 1 *An equilibrium* μ *is stochastically stable if and only if no other equilibrium has lower stochastic potential.*

[illegible]

[illegible] $I = I^*$ [illegible] $y = V(I^*) - x$ [illegible] B [illegible] x [illegible] $x < x^{NBS}$ [illegible] $V(I^*) = ;$ [illegible] x [illegible] $x > x^{NBS}$; [illegible] x [illegible] $r(x)$; [illegible] B [illegible] A [illegible] $(I; V(I) - -; -)$ [illegible] $(I^*; V(I^*) - -; -)$; [illegible] $r(x)$ [illegible] $(I^*; V(I^*) - x; x)$ [illegible] $(I; V(I) - -; -)$ [illegible]

$r\ x() > r\ x()$ $x > x^L$

$x^M > x^{NBS}$

$r\ x^L() \geq r\ x^L -- ()$

$$- \longrightarrow\ - \longrightarrow \ldots \longrightarrow x^L -- \longrightarrow x^L;$$

$r\ x^L() \geq r\ x^L -- ()$

$$x^L \longrightarrow - \longrightarrow\ - \longrightarrow \ldots \longrightarrow x^L --.$$

Appendix: Proofs

Lemma 1 *Let* $z_1 < z_2 ::: < z$ *be demands in* $D\ I()$ *for some* $I \in \Psi$. *Assume that the set of demands following* I *for agents in the relevant population is* $\{z_l\}_l$

Lemma 4 *Let* μ' *(*‰ $\mu'( ) = \{I'; y'; x'\}$*) be an equilibrium. If* $I \neq I'$ *and* $y - I \geq y' - I'$*, then the population can get from* μ' *to an equilibrium* μ *with* ‰ $\mu( ) = \{I; y; x\}$ *through a sequence of single mutation transitions.*

$I;V\ I\ (\,)--(\,)$

$\{I^*;y;x\}$ $x \leq x^L$

$V\ I^*(\,) - x^L(\,)\ N - I(\,) = N - I^* \leq V\ I^*(\,) - x(\,)\ N - I(\,) = N - I^*$

A $x(\,)$ µ i

A

$I; V - -; -)$

B

$x^L \in \{x^M; x^M - -\}$:

Lemma 5 *The number of mutations required to get from an equilibrium with outcome* $(I^*; y; x)$ *with* $x \leq x^L$ *to an equilibrium with outcome* $(I; V - -; -)$ *is* $r(x) = \min\{r | r > N(1 - \frac{\hat{V} - \delta - \hat{I} + I^*}{V^* -})\}$.

r B

$$\frac{N - r}{N}(V^* - x) - I^* < V - I - -;$$

r □

Lemma 6

(i) If µ *is an equilibrium with outcome* $(I^*; y; x)$ *and* $x < \min\{x^M; x^{NBS}\}$;
the easiest transition away from ‰ µ() Sdo2655la9e(0082TΩ0.9482 2 2 2F7283m)-343(with)-38326

[illegible] $x > x^L$ [illegible] x [illegible] x^L [illegible] x^M [illegible] $x^M < x^{NBS}$; [illegible] $r(x) \geq r(x)$ [illegible] $V^* - x^M -- \leq V --- I$ □

[illegible] x [illegible] $x^M > x^{NBS}$ [illegible] $I; V --; -)$ [illegible] $I^*; y$ $-; x--)$ [illegible]

[illegible] $I^*; y; x)$ [illegible] y [illegible] $V^* - x)$ [illegible] $I^*; y'; x')$ [illegible]

Lemma 7 *From an outcome $(I^*; y; x)$ the easiest transition in which investment is at all times e–cient, but which ends with difierent demands, is to an outcome $(I^*; y'; x')$ where x [illegible] $x --; x$ [illegible] $-; -;$ or $V^* --$.*

[illegible] □

[illegible] $-;$ [illegible] $-$

Lemma 8

(i) Moving from x to $x --$ takes N [illegible] $-\delta$ [illegible] mutations to pop A.

(ii) Moving from x to x $-$ takes N [illegible] $\frac{V^* - -\delta}{V^* -}$ mutations to pop B.

(iii) Moving from x to $-$ takes N $\frac{V^* -}{V^* - \delta}$ mutations to pop B.

(iv) Moving from x to $V^* --$ takes N $\frac{}{V^* - \delta}$ mutations to pop A.

[illegible] N □

Lemma 9

(i) If $- < x < V^* --$, then moving from x to $x --$ takes fewer mutations than moving from x to $-$, and moving from x to x $-$ takes fewer mutations than moving from x to $V^* --$.

(ii) If x [illegible] *– then moving from x to*

[illegible] $x^L > x^{NBS}$ [illegible] Θ^1 [illegible] Γ^1 [illegible] $x < x^{NBS}$ [illegible] $\mu;\mu'_{+\delta}$ [illegible] $x^{NBS} < x \leq x^L$ [illegible] $\mu;\mu'_{-\delta}$ [illegible] $r(x) < r(x-)$ [illegible] $x < x^{NBS}$ [illegible] $r(x) > r(x-)$ [illegible] $x > x^{NBS}$ □

[illegible] A [illegible] B [illegible] B [illegible] I [illegible] A [illegible] y [illegible]

Lemma 11 *Let surplus be divided by the ultimatum game. The component with the subgame perfect outcome, $(I^H; V^H - x^{\max}(I^H); x^{\max}(I^H))$, is a subset of the unique locally stable set.*

[illegible] $\exists\mu^H$ [illegible] $\mu^H \in T(\mu)$ [illegible] μ [illegible] $\%(\mu^H) = \{(I^H; V^H - x^{\max}(I^H); x^{\max}(I^H))\}$ [illegible] μ [illegible] $\%(\mu) = \{(I; V(I) - x; x)\}$ [illegible] A [illegible] $x^{\max}(I')$ [illegible] $I' \neq I$ [illegible] $x^{\max}(I)$ [illegible] I [illegible] B [illegible] $x^{\max}(I)$ [illegible] B [illegible] A [illegible] I^H; [illegible]

□

Lemma 12 *Let surplus be divided by the ultimatum game. Agents in population A receive a payoff of at least $V^H - I^H - x^{\max}(I^H)$ in every equilibrium.*

[illegible] I^H [illegible] A [illegible] I^H □

Lemma 13 *Let surplus be divided by the 'ultimatum' game. If $V(I) - I - x \geq V^H - I^H - x^{\max}(I^H)$, then there exists an equilibrium μ such that $\mu \in \Theta^L$ and $\%(\mu) = (I; V(I) - x; x)$.*

6 References

[illegible] *Journal of Economic Theory* [illegible]

[illegible] *Economic Journal* [illegible]

[illegible] *Review of Economic Studies* [illegible]

[illegible] *Games and Economic Behavior* [illegible]

[illegible] *Quarterly Journal of Economics* [illegible]

[illegible]

[illegible] *American Economic Review* [illegible]

[illegible] *Theoretical Population Biology* [illegible]

[illegible] *Games and Economic Behavior* [illegible]

[illegible] *Journal of Political Economy* [illegible]

[illegible] *Econometrica* [illegible]

[illegible] *Journal of Law, Economics and Organization* [illegible]

[illegible] *Markets and Hierarchies: Analysis and Antitrust Implications,* [illegible]

[illegible] *Econometrica* [illegible]

[illegible] *Journal of Economic Theory* [illegible]

[illegible] *Review of Economic Studies* [illegible]