

# GRAPH NEURAL NETWORKS FOR CAUSAL INFERENCE UNDER NETWORK CONFOUNDING

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**Abstract.**    s p p    s    s | n<sup>e</sup> n<sup>e</sup>    o    s    on | n<sup>e</sup>    s

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o    p o<sup>e</sup> n    | o    o<sup>e</sup>    s    n    s    |    on n o<sup>e</sup>    e<sup>n</sup>    o<sup>e</sup>    p l<sup>e</sup>    o p<sup>e</sup>

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on<sup>e</sup> n n    n<sup>e</sup>    s    e<sup>s</sup>    now n | o<sup>v</sup>    e<sup>s</sup> s on | n<sup>e</sup>    on o<sup>i</sup>

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o<sup>i</sup> n on s<sup>e</sup>    W<sup>e</sup>    s    o<sup>v</sup>    p    n<sup>e</sup>    |    n<sup>e</sup>    s<sup>e</sup>    s    e<sup>s</sup>    s<sup>e</sup>    ||    s    e<sup>s</sup>    o

s    o<sup>e</sup>    W<sup>e</sup>    e<sup>s</sup> s on | n<sup>e</sup>    s on<sup>e</sup> n n    e<sup>s</sup>    s    |    s    n<sup>e</sup>    s<sup>e</sup>    n

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e<sup>s</sup> on s<sup>e</sup>    e<sup>s</sup>    e<sup>s</sup>    o<sup>e</sup> |    s | o<sup>v</sup>    e<sup>s</sup> s on | s    e<sup>s</sup>    e<sup>s</sup>    s

# 1 Introduction

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## GNNs for Network Confounding

# Leung and Loupos

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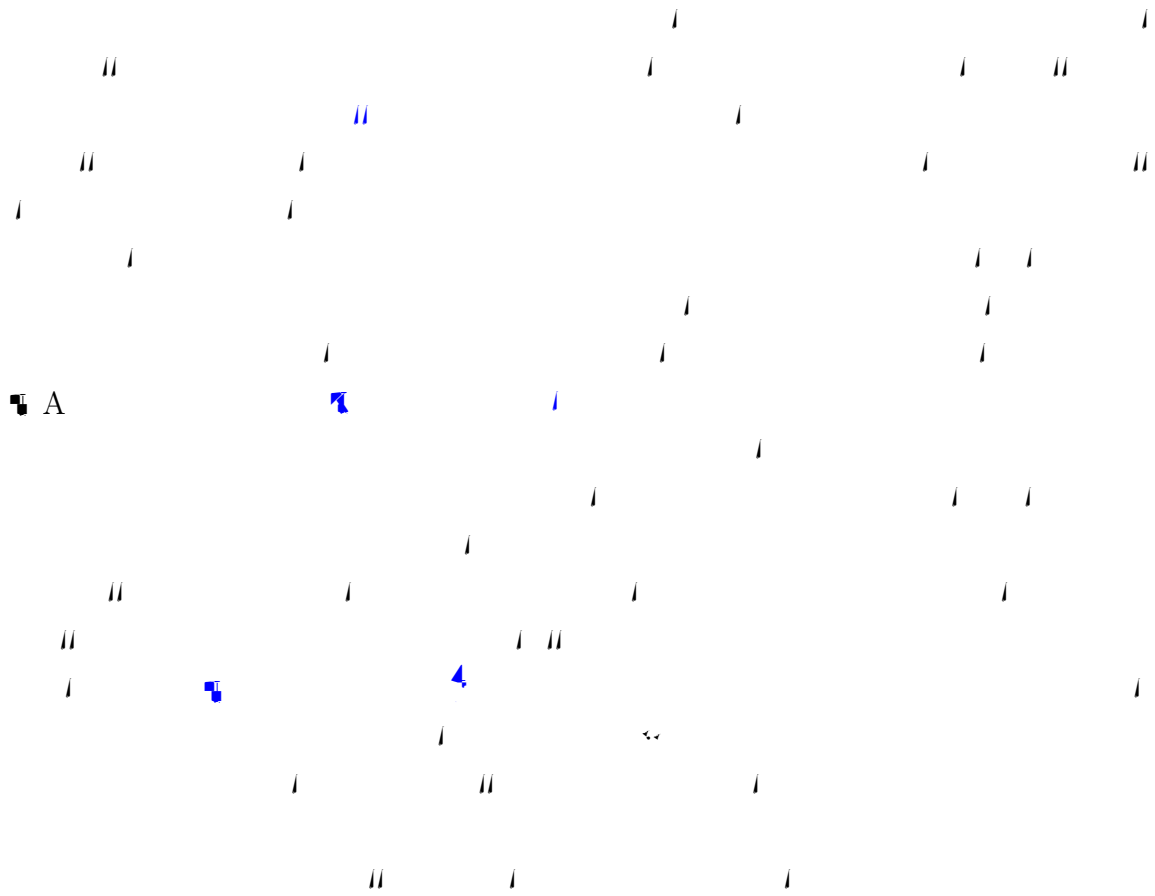
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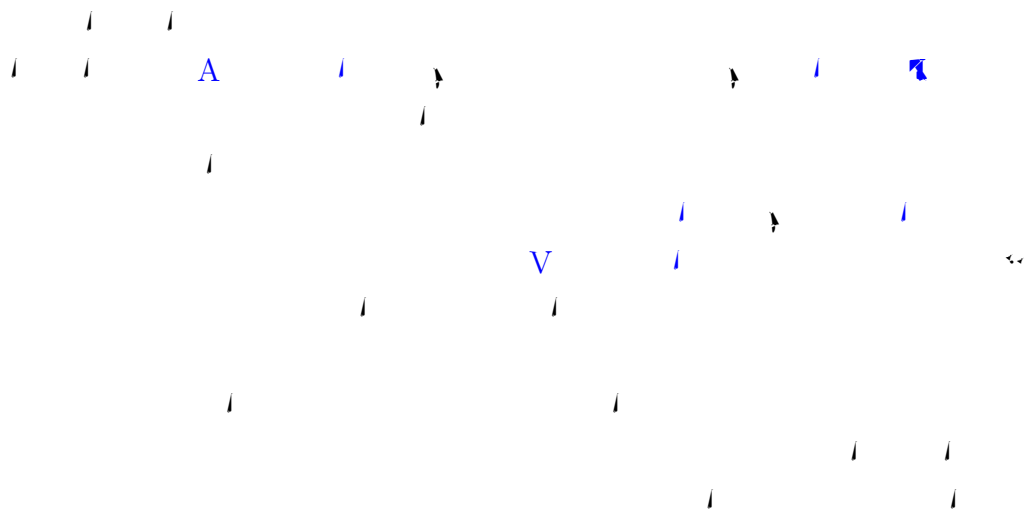
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# GNNs for Network Confounding



## 1.2 Related Literature





# GNNs for Network Confounding

$i \in \mathcal{N}_n = \{1, \dots, n\}$        $\mathbf{K} = \{1, \dots, K\}$   
 $A_{ij} \in \{0, 1\}$        $\mathbf{n}(i, 1)$

## 2 Setup

$\mathcal{N}_n = \{1, \dots, n\}$        $A \in \mathbb{R}^{n \times n}$   
 $i \in \mathcal{N}_n$        $(\mathbf{D}_i, \mathbf{X}_i) \in \mathbb{R}^{d_\varepsilon} \times \mathbb{R}^{d_\nu}$        $\mathbf{X}_i \in \mathbb{R}^{d_x}$

$$Y_i = g_n(i, D, X, A, \varepsilon) \quad D_i = h_n(i, X, A, \nu)$$

$\mathbf{X} = (\mathbf{X}_i)_{i=1}^n$        $(g_n, h_n)_{n \in \mathbb{N}}$   
 $g_n: \mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$        $h_n: \mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$   
 $(Y, D, X, A)$        $(A, X, \varepsilon, \nu)$

$(A, X, \varepsilon, \nu)$        $h_n(\cdot)$        $g_n(\cdot)$   
 $h_n(\cdot)$        $Y_i$        $D_i$

$E = \mathbb{P}_1$

$$Y_i = \frac{\sum_{j=1}^n A_{ij} Y_j}{\sum_{j=1}^n A_{ij}} + \frac{\sum_{j=1}^n A_{ij} Z'_j}{\sum_{j=1}^n A_{ij}} + Z'_i + \varepsilon_i$$

$$Z_i = (D_i, \mathbf{X}'_i)$$

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$$Y = \frac{1}{1+Z} + \sum_{k=0}^{\infty} \tilde{A}^{k+1} Z + \sum_{k=0}^{\infty} \tilde{A}^k \epsilon.$$

$$Y_i = g_n(i, D, X, A, \epsilon)$$

E p 2

$$D_i = 1 + \frac{\sum_{j=1}^n A_{ij} D_j}{\sum_{j=1}^n A_{ij}} + \frac{\sum_{j=1}^n A_{ij} Z'_j}{\sum_{j=1}^n A_{ij}} + Z'_i + \epsilon_i$$

U

$$(X, A, \nu)$$

D

$$D_i = h_n(i, X, A, \nu)$$

$$D_j = 1$$

$$D_i = h_n(i, X, A, \nu)$$

E p 2

$$D_i = i$$

$$Y_i$$

$$Y_i = g_n(D_{N(i,K)}, i, D_i = 1, W'_i, \dots)$$

W<sub>i</sub>

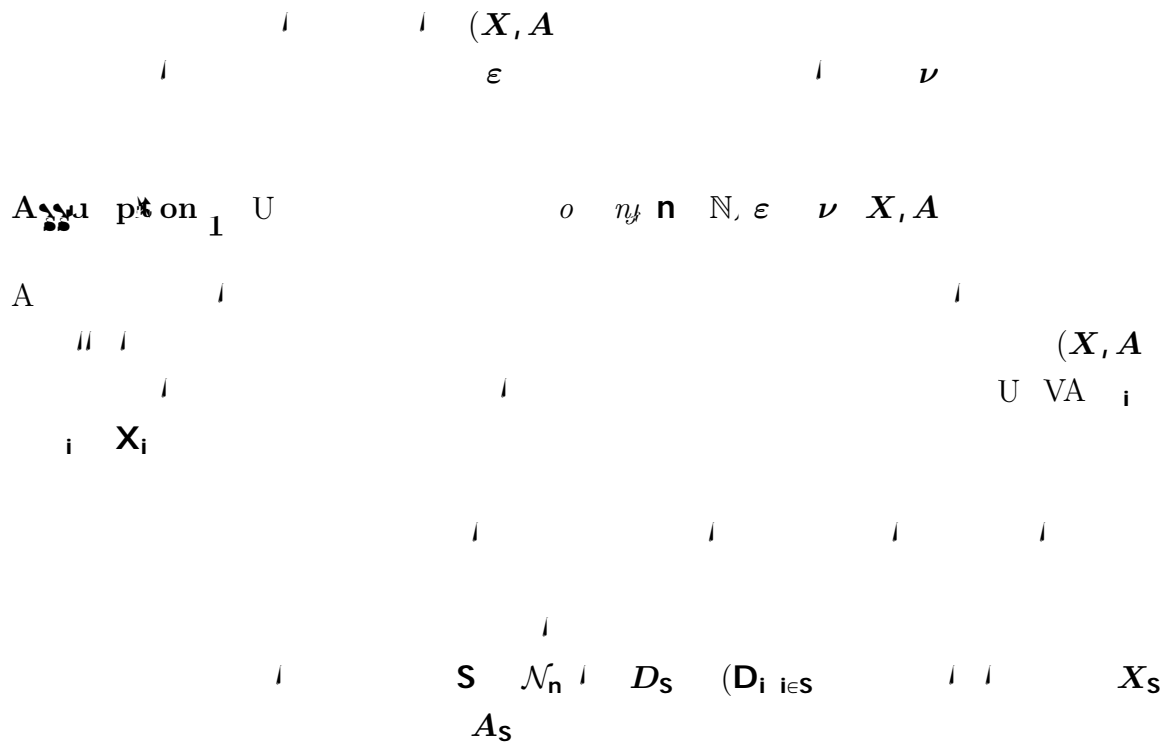
$$(X, A, K)$$

$$Y_i(d) = g_n(i, d, X, A, \epsilon)$$

$$Y_i(d) = \dots D_i$$



# GNNs for Network Confounding



$\mathcal{D}_i$   
 $E_p$   
 $C_0$   
 $D_{\mathcal{N}(i,K)}$   
 $c_1$

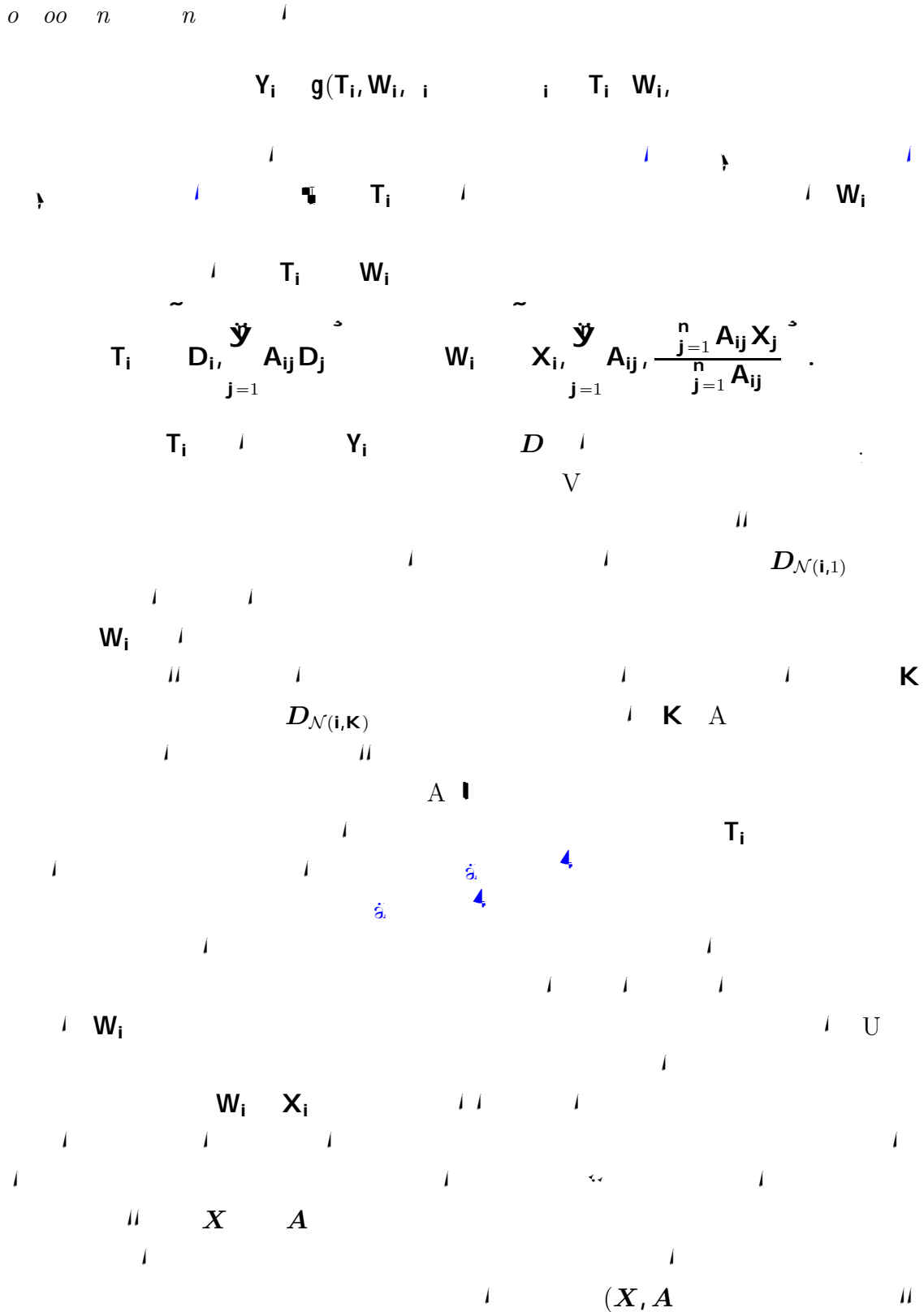
$Y_i$   
 $W_i$   
 $X_i$

$\sup_n \mathbb{E} \left[ \sum_{i=1}^n \left( \frac{Y_i}{W_i} - g(D_i, X_i) \right)^2 \right]$   
 $\sup_n \mathbb{E} \left[ \sum_{i=1}^n \left( \frac{Y_i}{W_i} - g(D_i, X_i) \right)^2 \right]$

## 2.1 Related Literature

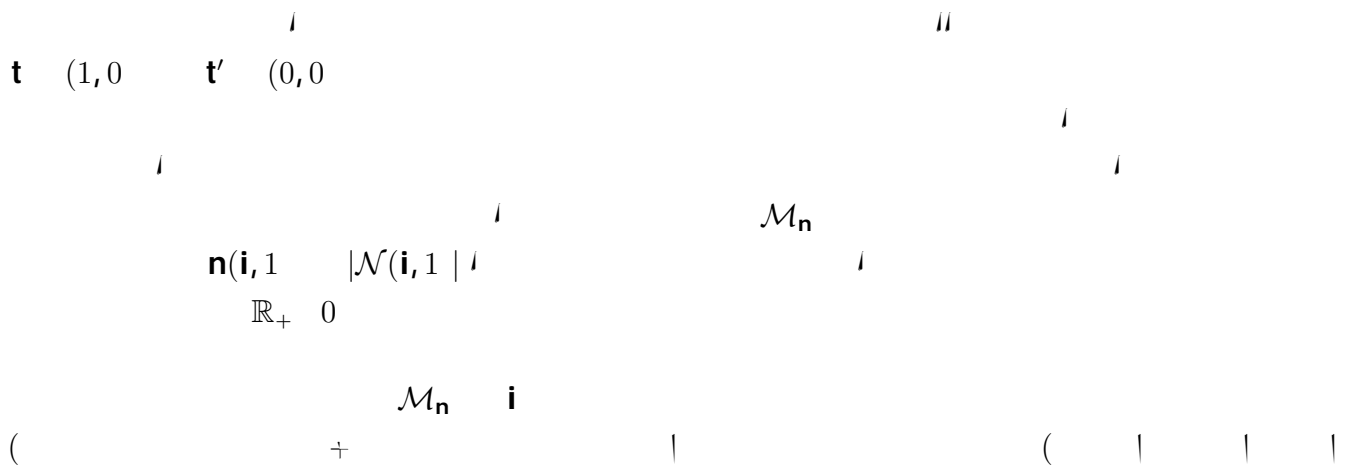
$U, V, A$   
 $Y_i = g(D_i, X_i) + \epsilon_i$   
 $T_i = f_n(i, D, A)$   
 $W_i = q_n(i, X, A)$   
 $f_n(\cdot)$   
 $q_n(\cdot)$   
 $n$

# GNNs for Network Confounding

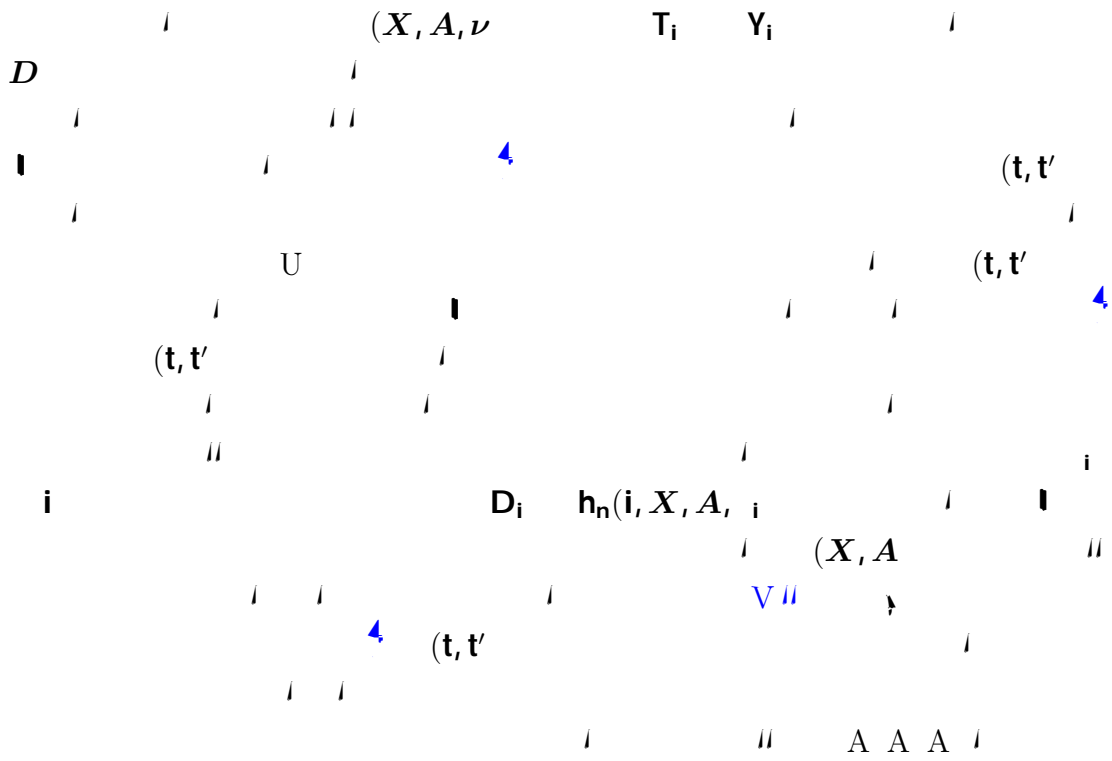




# GNNs for Network Confounding



Leung and Loupos



## GNNs for Network Confounding

$$\hat{\tau}_i(\mathbf{t}, \mathbf{t}') = \frac{1}{n} \sum_{i \in \mathcal{T}_t} \frac{Y_i - \hat{\mu}_t(\mathbf{i}, \mathbf{X}, \mathbf{A})}{\hat{\rho}_t(\mathbf{i}, \mathbf{X}, \mathbf{A})} - \hat{\mu}_t(\mathbf{i}, \mathbf{X}, \mathbf{A}) - \frac{1}{n} \sum_{i \in \mathcal{T}_{t'}} \frac{Y_i - \hat{\mu}_{t'}(\mathbf{i}, \mathbf{X}, \mathbf{A})}{\hat{\rho}_{t'}(\mathbf{i}, \mathbf{X}, \mathbf{A})} - \hat{\mu}_{t'}(\mathbf{i}, \mathbf{X}, \mathbf{A}) .$$

### **3.1 Architecture**



## GNNs for Network Confounding

## Leung and Loupos

$$\begin{aligned}
 & \mathbf{X}_i \\
 & \Gamma(\boldsymbol{\mu}(\cdot, \cdot), \Sigma(\cdot, \cdot), \min(\cdot, \cdot), \max(\cdot, \cdot)) \\
 & \Gamma(\cdot) \Gamma_1(\cdot) \\
 & \Gamma_1(\boldsymbol{\mu}(\cdot, \cdot), \Sigma(\cdot, \cdot), \min(\cdot, \cdot), \max(\cdot, \cdot)) \\
 & \mathbf{n}(\mathbf{i}, 1) \\
 & \Gamma_1(\cdot) \\
 & \mathbf{S}(\cdot, \frac{\log(|\cdot| + 1)}{\cdot}, \frac{1}{\mathbf{n}} \sum_{i=1}^{\mathbf{y}} \log \sum_{j=1}^{\mathbf{y}} \mathbf{A}_{ij} + 1, \cdot) \quad | \cdot, 1, 1. \\
 & 1 \\
 & 0
 \end{aligned}$$

## GNNs for Network Confounding

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$$\hat{\mathbf{f}}_{\text{GNN}} = \underset{\mathbf{f} \in \mathcal{F}_{\text{GNN}}(\mathbf{L})}{\text{argmin}} \|\mathbf{y} - \mathbf{f}\|_2$$

A

**f**  $\mathcal{F}_{\text{GNN}}(\mathbf{L}$



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$$\begin{aligned}
 \mu_{t,t'}(\mathbf{i}) &= \frac{1}{\mathbf{p}_t(\mathbf{i}, \mathbf{X}, \mathbf{A})} \sum_{\mathbf{d}} \mathbf{Y}_i(\mathbf{d} | \mathbf{p}, \mathbf{X}, \mathbf{A}) \mu_{t,t'}(\mathbf{i}, \mathbf{X}, \mathbf{A}, \mathbf{d}) \\
 &= \frac{1}{\mathbf{p}_{t'}(\mathbf{i}, \mathbf{X}, \mathbf{A})} \sum_{\mathbf{d}} \mathbf{Y}_i(\mathbf{d} | \mathbf{p}, \mathbf{X}, \mathbf{A}) \mu_{t,t'}(\mathbf{i}, \mathbf{X}, \mathbf{A}, \mathbf{d}) \quad (t, t', \mathbf{i})
 \end{aligned}$$

$\mathbf{i} \in \mathcal{M}_n$

$$\frac{1}{\mathbf{m}_n} \sum_{\mathbf{i} \in \mathcal{M}_n} \mathbf{Y}_i(\mathbf{d} | \mathbf{p}, \mathbf{X}, \mathbf{A})$$

**Assumption**  $\mathbf{M}$   $n \times p$   $4 \times 4$   $o(n)$

$n \in \mathbb{N}$ ,  $\mathbf{i} \in \mathcal{M}_n$ ,  $n \times d$   $0, 1^n$ ,  $\|\mathbf{Y}_i(\mathbf{d} | \mathbf{p}, \mathbf{X}, \mathbf{A})\| \leq \mathbf{M}$

$\mathbf{p}_t, \mathbf{p}_{t'} \in (0, 1)^u$ ,  $\hat{\mathbf{p}}_t(\mathbf{i}, \mathbf{X}, \mathbf{A}), \hat{\mathbf{p}}_{t'}(\mathbf{i}, \mathbf{X}, \mathbf{A}) \in \mathbb{R}^n$

# GNNs for Network Confounding

$$t, t' \left( \prod_{i=1}^n \right)$$

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$$\Lambda_n(s) = \int_{-2M}^{-M} \int_{-M}^M \int_{-M}^M \prod_{i=1}^n t_i^{t_i} \dots$$



# GNNs for Network Confounding



or  $\mathcal{D} = \{n \sim_{t,t'}(\mathbf{i}) \mid n \in \{t, t'\} \}$   $n$  on  $o_{t,t'}(\mathbf{i}) \mid \mathbf{i}(t, t')$   
 $\{Y_i \mid T_i = t, X_i\}$

$$\| \hat{\mathbf{X}} - \mathbf{A} \|_F^2$$

## 5 Approximate Sparsity

$$\mathbf{A} \mathbf{X}_{\mathcal{N}(i,L)} \mathbf{L}$$









## GNNs for Network Confounding

$$(W_i)_{i=1}^n \quad \nu \quad (i)_{i=1}^n$$

$$V_i(W, \nu; \dots) + \frac{\sum_{j=1}^n A_{ij} W_j}{\sum_{j=1}^n A_{ij}} + \frac{\sum_{j=1}^n A_{ij} X_j}{\sum_{j=1}^n A_{ij}} + X_j + i + \frac{\sum_{j=1}^n A_{ij} j}{\sum_{j=1}^n A_{ij}}$$

$$Y_i \quad V_i(Y, \varepsilon; y) \quad y \quad (0.5, 0.8, 10, 1) \quad D_i \quad 1 \quad V_i(D, \nu; d) \quad 0 \quad d \quad (0.5, 1.5, 1, 1)$$

$$D_i^0 \quad 1 \quad V_i(0, \nu; d) \quad 0$$

$$\frac{\sum_{j=1}^n A_{ij} j}{\sum_{j=1}^n A_{ij}} \quad A \quad A$$

$$T_i \quad t \quad | Y_i \quad T_i \quad t, X, A \quad n \quad 1000, 2000, 4000$$

### 6.2 Nonparametric Estimators

$$L \quad 1, \quad \Gamma_2(\dots)$$

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		$L = 1$			$L = 2$			$L = 3$		
$n$	$e \quad e$									
$H$		<hr/>								
$\hat{\tau}(1,0)$										
$Cl$										
$e \quad Cl$										
$W \hat{\tau}(1,0)$										
$W Cl$										
$W$										
$\parallel \quad Cl$										
$\parallel$										

• In situations, the estimator is  $\hat{\tau}(1,0) = 0$ , treated  $\approx$







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## **7.1 Comparison with He and Song (2024)**



# GNNs for Network Confounding

**n** 4413

|0.01, 0.99

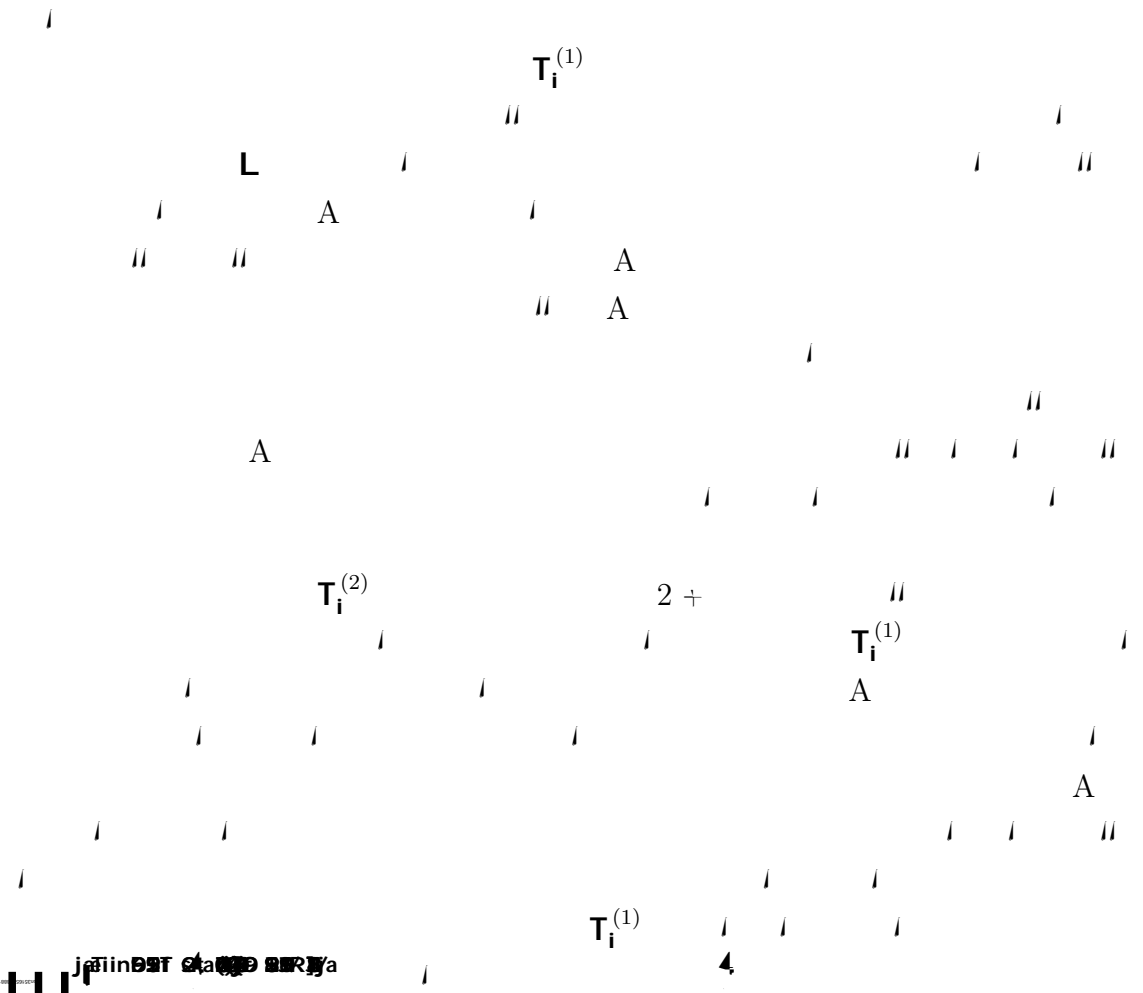
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$T_i^{(1)}$

	ADM	GNN			GLM		
		Layer	Layer	Layer	Order	Order	Order
Leader case							
$G_{ee}$	003	000	000	000	000	000	000
$G_{sc}$	00	00	00	00	00	00	00
$G_{all}$	003	00	00	00	00	003	00
Leader adopter case							
$G_{ee}$	0'3	00	00	00300	00	003	00
$G_{sc}$	0'	00	003	00	00	00	00
$G_{all}$	0'3	00	00	00	00	00	0'3000
Adopter case							
$G_{ee}$	0'	00	00	003003	00300	00300	0'00
$G_{sc}$	0'	00	003	00	00	00300	0'00
$G_{all}$	0'3	00	00	00	00	00	0'00

n = 4413

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misal sobe  
jainBT Stage 2 Rya



## A Additional Results on GNNs

$$\begin{aligned}
 & \frac{1}{m_n} \sum_{i \in \mathcal{M}_n} \|\hat{\mathbf{y}}_t(i, \mathbf{X}, \mathbf{A}) - \mathbf{p}_t(i, \mathbf{X}, \mathbf{A})\|^2 = o_p(n^{-1/2}). \\
 & \frac{1}{m_n} \sum_{i \in \mathcal{M}_n} \|\hat{\mathbf{y}}_t(i, \mathbf{X}, \mathbf{A}) - \mathbf{p}_t(i, \mathbf{X}_{\mathcal{N}(i,L)}, \mathbf{A}_{\mathcal{N}(i,L)})\|^2 = o_p(n^{-1/2}). \\
 & \|\hat{\mathbf{y}}_t(i, \mathbf{X}, \mathbf{A}) - \mathbf{p}_t(i, \mathbf{X}_{\mathcal{N}(i,L)}, \mathbf{A}_{\mathcal{N}(i,L)})\| \leq L \|\mathbf{X}_{\mathcal{N}(i,L)} - \mathbf{X}\| + \|\mathbf{A}_{\mathcal{N}(i,L)} - \mathbf{A}\|. \\
 & \frac{1}{n} \sum_{i=1}^n \|\hat{\mathbf{y}}_t(i, \mathbf{X}, \mathbf{A}) - \mathbf{p}_t(i, \mathbf{X}_{\mathcal{N}(i,L)}, \mathbf{A}_{\mathcal{N}(i,L)})\|^2 \leq C \frac{WL \log R}{n} \log n + \frac{\log \log n}{n} + \frac{1}{n^2}. \\
 & \frac{1}{n} \sum_{i=1}^n \|\hat{\mathbf{y}}_t(i, \mathbf{X}, \mathbf{A}) - \mathbf{p}_t(i, \mathbf{X}_{\mathcal{N}(i,L)}, \mathbf{A}_{\mathcal{N}(i,L)})\| \leq \frac{1}{n} e^{-\frac{1}{n}} \frac{W}{nR} + C.
 \end{aligned}$$



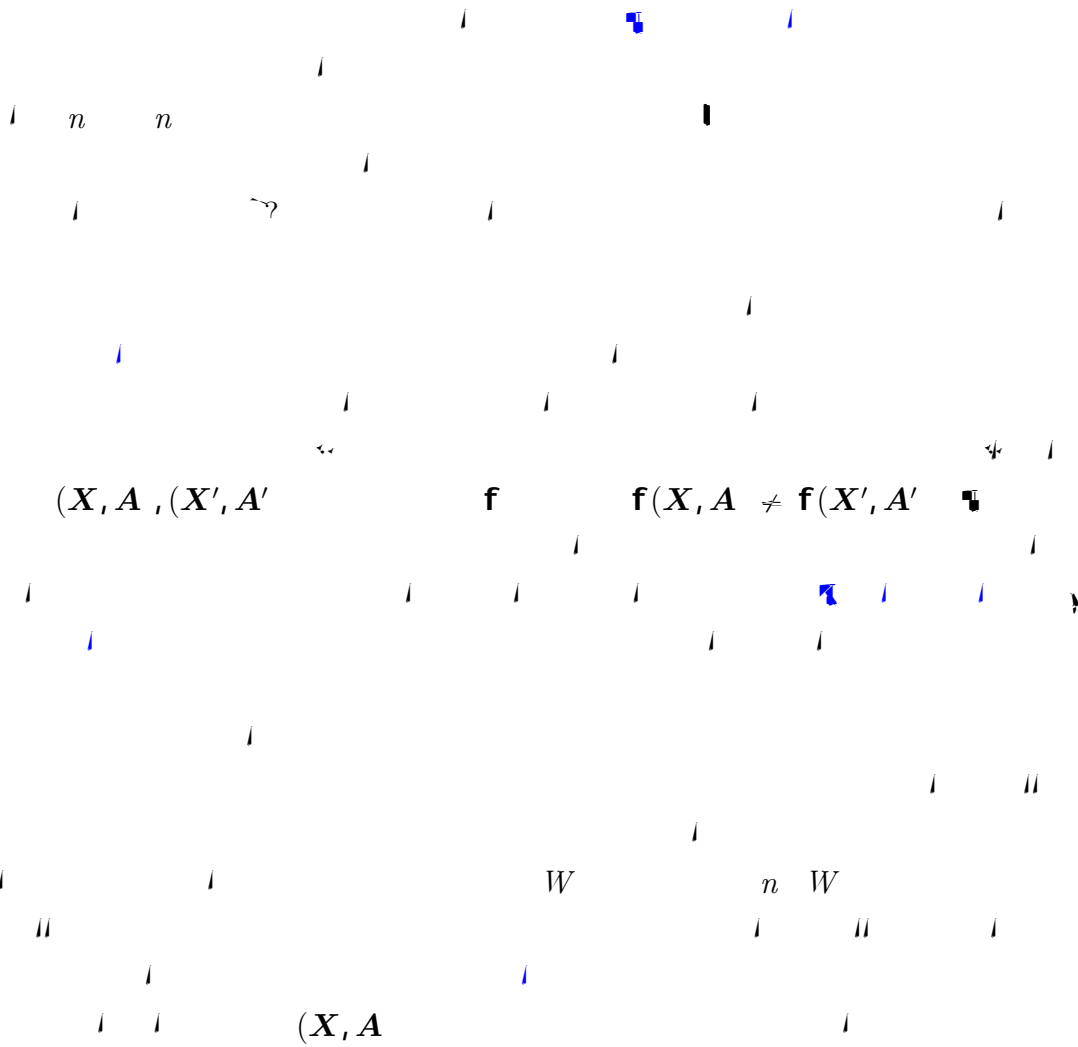
# GNNs for Network Confounding

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$W, R, L, n$  A

## A.1 WL Function Class



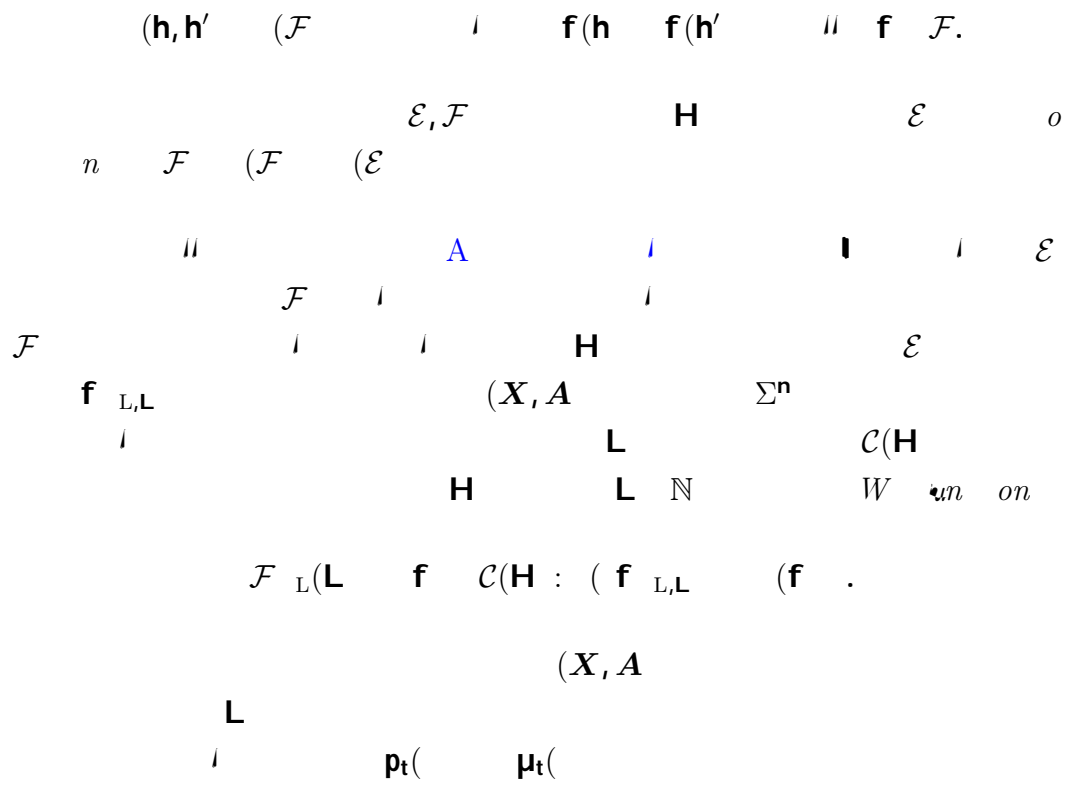
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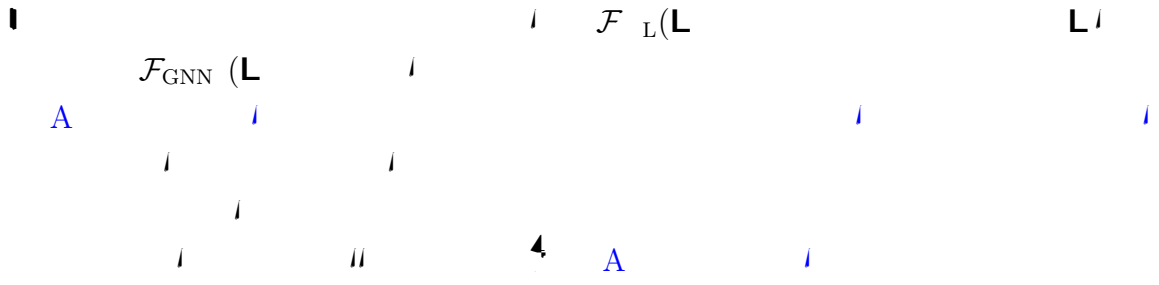
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# GNNs for Network Confounding

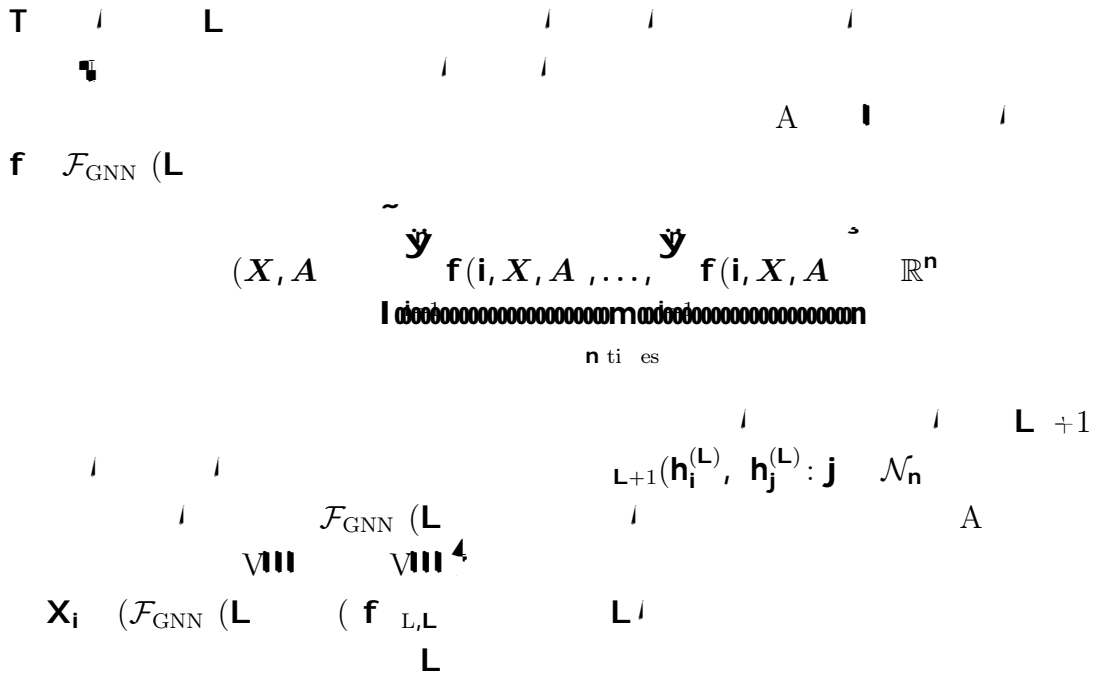
$H^2$



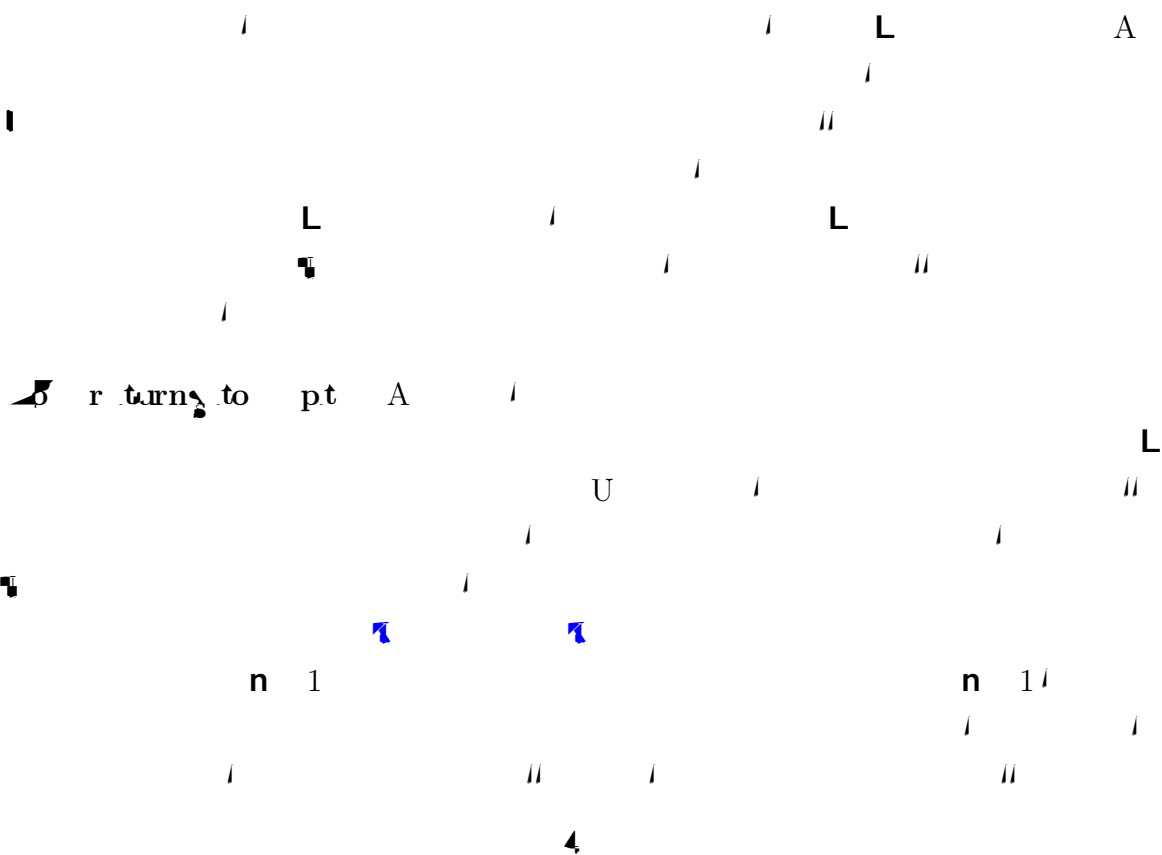
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# GNNs for Network Confounding



## A.2 Disadvantages of Depth





# GNNs for Network Confounding

## B Verifying §8 Assumptions

$$\begin{aligned}
 & \mathbb{P} \left( \max_{\mathbf{A}} \max_{\mathbf{s}} \left| \mathcal{N}_{\mathbf{A}}(\mathbf{i}, \mathbf{s}) \right| \geq C s^d \right) \\
 & \leq \mathbb{P} \left( \max_{\mathbf{A}} \max_{\mathbf{s}} \left| \mathcal{N}_{\mathbf{A}}(\mathbf{i}, \mathbf{s}) \right| \geq C s^d \right) \\
 & \leq \mathbb{P} \left( \max_{\mathbf{A}} \max_{\mathbf{s}} \left| \mathcal{N}_{\mathbf{A}}(\mathbf{i}, \mathbf{s}) \right| \geq C s^d \right)
 \end{aligned}$$

$$\mathbf{C} = 0 \quad d > 1$$







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$$D'_B = (D'_j)_{j \in B} \quad B \in \mathcal{N}_n \cup \{ \emptyset \}$$

$$p_t(i, X, A) = (D'_i + (D_i - D'_i) \mathbb{1}_{\{a, b\}}, V'_i + (V_i - V'_i) \mathbb{1}_{\{a, b\}}) \cdot (X, A) + \mathbb{1}_{\{a, b\}} \cdot (D_i - D'_i) \cdot (X, A) + (V_i - V'_i) \cdot (X, A) \cdot R_0$$

/

$$(D'_i \mathbb{1}_{\{a, b\}})$$

# GNNs for Network Confounding

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$$p_t(\mathbf{i}, \mathbf{X}, \mathbf{A}) = p_t(\mathbf{i}, \mathbf{X}_{\mathcal{N}(\mathbf{i}, r_\lambda(s+1))}, \mathbf{A}_{\mathcal{N}(\mathbf{i}, r_\lambda(s+1))}) \mid \mathbf{n}(\mathbf{s} + 1) \sim \mathbf{R}_0.$$

$$\begin{aligned} \mu_t(\mathbf{i}, \mathbf{X}, \mathbf{A}) &= \mathbb{E}[Y_i \mid \mathbf{i}, \mathbf{X}, \mathbf{A}] \\ \mathbf{B} &= \mathcal{N}(\mathbf{i}, \mathbf{s}) \\ Y_i' &= g_{\mathbf{n}(\mathbf{i}, \mathbf{s})}(\mathbf{i}, D_{\mathbf{B}}, X_{\mathbf{B}}, A_{\mathbf{B}}, \epsilon_{\mathbf{B}}) \end{aligned}$$

$$|Y_i \mathbf{1}_i(t) \mid \mathbf{X}, \mathbf{A} = |Y_i' \mathbf{1}_i(t) \mid \mathbf{X}, \mathbf{A} \mid \mathbf{n}(\mathbf{s} + \Lambda_{\mathbf{n}}(\mathbf{i}, \mathbf{s})) \mid \mathbf{n}(\mathbf{i}, \mathbf{s}) \mid \mathbf{n}(\mathbf{s})$$

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$|R_1|$  n

## GNNs for Network Confounding

$$|R_1| = \sum_{i \in \mathcal{N}_n} \Lambda_n(i, s) \mathbb{1}_i(s) \quad \blacksquare$$

$\blacktriangleright$   $\mathcal{C}^c = \{ (i, s) \mid \exists t' \neq t, \mathbb{1}_i(t') = 1, \mathbb{1}_i(t) = 0, \sum_{j=1}^n A_{ij} D_j \neq 0 \}$   
 $n \in \mathbb{N}, i \in \mathcal{N}_n, n \geq 1$

$$|Y_i| \mathbb{1}_i(t) - \mathbb{1}_i(t') |X, A| = C \sum_{i \in \mathcal{N}_n} \mathbb{1}_i(s)$$

**Proof.**  $\parallel$   $a, b, \dots, V_i = \sum_{j=1}^n A_{ij} D_j$   
 $V_i' = \sum_{j=1}^n A_{ij} D_j' \in \mathcal{C} \mid D_i = D_i', \mid V_i = V_i'$

$$|Y_i| \mathbb{1}_i(t) - \mathbb{1}_i(t') |X = x, A = a$$

$$|Y_i| \mathbb{1}_i(t) - \mathbb{1}_i(t') |C, X = x, A = a + C \in \mathcal{C}^c \mid X = x, A = a$$

$$\mid C = 0 \mid A = A$$

$$\mathbb{1}_i(t) - \mathbb{1}_i(t') \mid D_i = |a, b, V_i|, \quad \mathbb{1}_i(t') - \mathbb{1}_i(t) \mid D_i' = |a, b, V_i'|, \dots$$

U  $\in \mathcal{C}$

$$\mathbb{1}_i(t) - \mathbb{1}_i(t') \mid D_i = |a, b, V_i|, \quad \mathbb{1}_i(t') - \mathbb{1}_i(t) \mid D_i' = |a, b, V_i'| + (V_i - V_i') \mid D_i'$$



## GNNs for Network Confounding

$$\begin{aligned}
 & A \quad \left( \mathbf{Z}_i^{(s/2, \cdot)} \right)_{i \in H} \quad \left( \mathbf{Z}_j^{(s/2, \cdot)} \right)_{j \in H^1} \quad \mathcal{F}_n \\
 & \left| \left( \cdot, \mathcal{F}_n \right) \right| \quad \left| \left( \cdot^{(s/2)}, \mathcal{F}_n \right) \right| + \left| \left( \cdot^{(s/2)}, \cdot^{(s/2)} \mathcal{F}_n \right) \right| \\
 & 2 \|\mathbf{f}'\|_\infty \left| \left( \cdot^{(s/2)} \right) \mathcal{F}_n \right| + 2 \|\mathbf{f}\|_\infty \left| \left( \cdot^{(s/2)} \right) \mathcal{F}_n \right| \\
 & 2 \mathbf{h} \|\mathbf{f}'\|_\infty \left( \mathbf{f} + \mathbf{h}' \|\mathbf{f}\|_\infty \right) \left( \mathbf{f}' \max_{i \in \mathcal{N}} \right)
 \end{aligned}$$

# Leung and Loupos

$$\mathcal{L}_h = \mathcal{L}_{h'} \quad \mathbf{s} = 0 \quad (\mathbf{H}, \mathbf{H}') \quad \mathcal{P}_n(\mathbf{h}, \mathbf{h}'; \mathbf{s})$$

$$\mathbf{Y}_i^{(s)} = \mathbf{g}_{n(i,s)}(\mathbf{i}, \mathbf{D}_{\mathcal{N}(i,s)}, \mathbf{X}_{\mathcal{N}(i,s)}, \mathbf{A}_{\mathcal{N}(i,s)}, \boldsymbol{\varepsilon}_{\mathcal{N}(i,s)})$$

$$\mathbf{f}(\mathbf{Y}_i)_{i \in \mathcal{H}} = \mathbf{f}'(\mathbf{Y}_i)_{i \in \mathcal{H}'} \quad \mathbf{s} = 0 \quad \mathbf{f}(\mathbf{Y}_i^{(s)})_{i \in \mathcal{H}} = \mathbf{f}'(\mathbf{Y}_i^{(s)})_{i \in \mathcal{H}'}$$

A

$$\begin{aligned} & \left| \left( \mathbf{f}, \mathcal{F}'_n \right) \right| \left| \left( \mathbf{f}^{(s/2)}, \mathcal{F}'_n \right) \right| + \left| \left( \mathbf{f}^{(s/2)}, \mathcal{F}'_n \right) \right| \left| \left( \mathbf{f}, \mathcal{F}'_n \right) \right| \\ & 2 \|\mathbf{f}'\|_\infty \|\mathbf{f}\|_\infty \|\mathbf{f}^{(s/2)}\|_\infty + 2 \|\mathbf{f}\|_\infty \|\mathbf{f}'\|_\infty \|\mathbf{f}^{(s/2)}\|_\infty \\ & 2 \|\mathbf{h}\| \|\mathbf{f}'\|_\infty (\|\mathbf{f}\|_\infty + \|\mathbf{h}'\| \|\mathbf{f}\|_\infty) (\mathbf{f}' \max_{i \in \mathcal{N}_n} \|\mathbf{Y}_i - \mathbf{Y}_i^{(s/2)}\|) \mathcal{F}'_n \\ & 2 \|\mathbf{h}\| \|\mathbf{f}'\|_\infty (\|\mathbf{f}\|_\infty + \|\mathbf{h}'\| \|\mathbf{f}\|_\infty) (\mathbf{f}' \mathbf{n}(\mathbf{s}^2)) \end{aligned}$$

$$\left| \left( \mathbf{f}, \mathcal{F}'_n \right) \right| \left| \left( \mathbf{f}^{(s/2)}, \mathcal{F}'_n \right) \right| + \left| \left( \mathbf{f}^{(s/2)}, \mathcal{F}'_n \right) \right| \left| \left( \mathbf{f}, \mathcal{F}'_n \right) \right|$$

A







## GNNs for Network Confounding

$\hat{\rho}_t(\mathbf{i}, \mathbf{X}, \mathbf{A})$

$$\Delta_{\mathbf{i}}(t) = (\hat{\rho}_t(\mathbf{i}) - \mu_t(\mathbf{i})) \mathbf{p}_t(\mathbf{i}) \mathbf{1}_{\mathbf{i}}(t)$$

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# GNNs for Network Confounding

Bronstein, M. <https://towardsdatascience.com/do-we-need-deep-graph-neural-networks>  
Feb 2020

→ Brain, Confounding,  $X$ ,  $n$ ,  $X$ ,  $\theta$

Confounding, Brain,  $A$ ,  $V$

Confounding, DC, DE, Data,  $ono$ ,  $ou n$

Confounding, DB,  $A$ ,  $V$

Donor,  $ou n$ ,  $A$ ,  $n$ ,  $A$ ,  $o$

Donor, FC,  $ou n$ ,  $ono$ ,  $5$ ,  $4$

Donor, Confounding, Y, B,  $n$ ,  $X$ , Br,  $ono$

En, C,  $U$ ,  $A$ ,  $X$ ,  $n$ ,  $X$ ,  $\theta$ ,  $5$

Frr,  $A$ ,  $X$ ,  $n$ ,  $X$ ,  $\theta$

→  $ono$



# GNNs for Network Confounding

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100





## GNNs for Network Confounding

o  $m$   $M$   $n$   $X$   $n$

