

SYMPLECTIC MAPS

$$\begin{aligned}
 & \text{symplectic} \\
 & q, p, \\
 & = p \wedge q. \tag{1}
 \end{aligned}$$

$$\begin{aligned}
 & (q_i, p_i), i=1, \dots, n,
 \end{aligned}$$

$$\begin{aligned}
 & (v, w) \in \{\nabla, \nabla^*, F(q, p)\}
 \end{aligned}$$

$$F = q' + p' + tH(q, p') \quad (2)$$

$$q' = q + t \frac{H}{p'}(q, p'), \quad p' = p - t \frac{H}{q}(q, p'). \tag{4}$$

S \rightarrow S

$$\begin{aligned} p' &= \frac{\partial H}{\partial q}, \quad q' = \frac{\partial H}{\partial p}, \quad H = K(p) + V(q) \\ &\text{S} = \left(\begin{array}{cc} 0 & I_n \\ -I_n & 0 \end{array} \right), \quad \text{J} = \left(\begin{array}{cc} 0 & -I_n \\ I_n & 0 \end{array} \right). \end{aligned}$$

The Symplectic Group

$$\begin{aligned} z_{t+1} &= f(z_t) = \{z_t, z_{t+1}, \dots\} \\ f &: \mathbb{R}^{2n} \rightarrow \mathbb{R}^{2n}, \quad M = \prod_t Df(z_t). \\ f &: \mathbb{R}^{2n} \rightarrow \mathbb{R}^{2n}, \quad M \in \mathbb{R}^{2n \times 2n}, \quad M^T J M = J. \\ &\text{Sp}(2n). \quad n(2n+1)- \\ JS &= \{S \in \text{Sp}(2n) : S^T J S = J\} \\ JS &= \{S \in \text{Sp}(2n) : S^T J S = J\} \\ &\text{Sp}(2n) = \{M \in \text{GL}(2n) : M^T J M = J, \\ &M^T M = I, \quad M^{-1} = M^T\}, \\ &\{M \in \text{GL}(2n) : M^T J M = J, \\ &M^T M = I, \quad M^{-1} = M^T\}. \\ &\{M \in \text{GL}(2n) : M^T J M = J, \\ &M^T M = I, \quad M^{-1} = M^T\}. \\ &\{M \in \text{GL}(2n) : M^T J M = J, \\ &M^T M = I, \quad M^{-1} = M^T\}. \\ &\{M \in \text{GL}(2n) : M^T J M = J, \\ &M^T M = I, \quad M^{-1} = M^T\}. \end{aligned}$$

- *hyperbolic*,
- *hyperbolic with reflection*,
- *elliptic*, $\lambda = 2$,
- *Krein quartet*.

$$Q = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 0 & 40 & 0 & 1 \\ 0 & 0 & 0 & 0.01 \end{pmatrix} = \begin{pmatrix} 4 & 0 & 0 & 0 \\ 0 & -2 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1.142 \end{pmatrix} \approx 10^{-10}$$

Let $\mathbf{v} = (v_1, v_2)$ be a vector in \mathbb{R}^2 . Then \mathbf{v} is called a **Lyapunov vector** if there exist $m, n \in \mathbb{N}$ such that $(0) \neq n$ and $m \leq v_1 \leq n$.

Let $\mathbf{v} = (v_1, v_2)$ be a vector in \mathbb{R}^2 . Then \mathbf{v} is called a **Lyapunov vector** if there exist $C > 0$, $D > 0$, $n \in \mathbb{N}$ and $(1) < \frac{v_1}{v_2} < (n+1)$.

Let $\mathbf{v} = (v_1, v_2)$ be a vector in \mathbb{R}^2 . Then \mathbf{v} is called a **Lyapunov vector** if there exist $n \in \mathbb{N}$ and $(1, 1) < \frac{v_1}{v_2} < (n+1)$.

Let $\mathbf{v} = (v_1, v_2)$ be a vector in \mathbb{R}^2 . Then \mathbf{v} is called a **Lyapunov vector** if there exist $n \in \mathbb{N}$ and $(1, 1) < \frac{v_1}{v_2} < (n+1)$ beyond all orders.

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Let $\mathbf{v} = (v_1, v_2)$ be a vector in \mathbb{R}^2 . Then \mathbf{v} is called a **Lyapunov vector** if there exist $n \in \mathbb{N}$ and $(1, 1) < \frac{v_1}{v_2} < (n+1)$ such that \mathbf{v} is a cantorus.

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See also Aubry–Mather theory; Cat map; Chaotic dynamics; Constants of motion and conservation laws; Ergodic theory; Fermi acceleration and Fermi map; Hamiltonian systems; Hénon map; Horseshoes and hyperbolicity in dynamical systems; Lyapunov exponents; Maps; Measures; Melnikov method; Phase space; Standard map

Further Reading

- Arnold, V. I. 1989. *Mathematical Methods of Classical Mechanics*, 2nd edn. Springer.
- Bonetto, R., L. S. Chua, and S. S. P. Yip. 1998. *Beam Dynamics: A New Attitude and Framework (The Physics and Technology of Particle and Photon Beams)*, 1st edn. Springer.
- Chau, K. W., and J. M. Greene. 2001. *Symplectic Twist Maps: Global Variational Techniques*, 1st edn. Springer.

Manuscript Queries

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Page	Query Number	Query
		No Query