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Ensemble-based estimates of eigenvector error for empirical covariance matrices

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[4 o 27 m r 2016 r o 28 r 2018 44 o 6 r 2018]

1. Introduction

4 r ro r of 4o r 4 m r 4 r 4 r o 4 m m 4 , ro 4 (, 1991 A r o , 2003 ., 2009 o & o , 2012) ro 4or r-

o . or m , o o from (1.5) 4 g g o

$$E[\ ] +$$

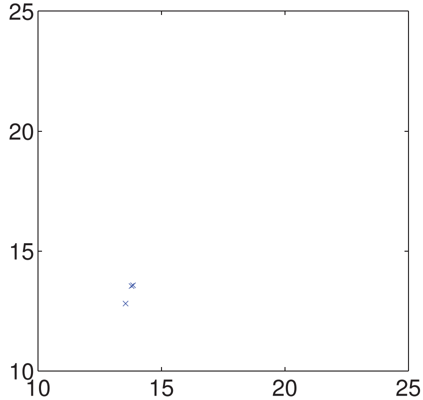
Assumption 2.2 [illegible] of [illegible] for [illegible]

$$(\ , \ ) \quad \frac{3^7 [\ \rho(\lambda\ )]^5}{32\pi^3} [\ \ , \langle$$

2.3 [illegible] 2 [illegible]

[illegible] o (2.2) g [illegible] m o 4 m for [illegible]

o o 4 (r ., 2017 T or ., 2017). fi r ork, o r g o o for gr mor 4om 4 n 4 r of 4 4om ork r rg o of ork or g 4 r for gr (rk ., 2001 o ., 2001 g ., 2003 orogo ., 2003 B 4- org & k , 2011 o o, 2013 Z g ., 2014 T or ., 2016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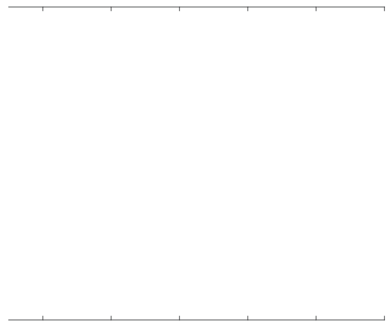

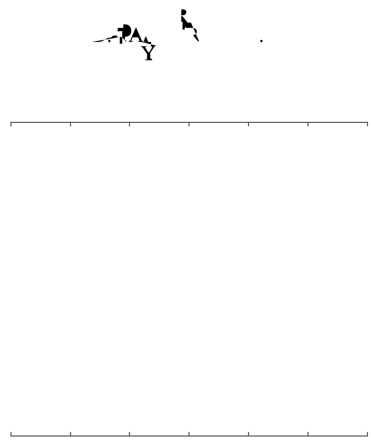

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A. Derivation of main result 1

A. , ro m (1.1) m of r - g o r g g . B o g o, o o k o g of , m g r o of g . g g mm o o o r o

$$+ \quad , \tag{A.1}$$

$$+ \sum_{+1}^{1} \frac{\lambda \lambda}{(\lambda \quad \lambda)^2}, \tag{A.2}$$

$$+ \sum_{+ \; 1} \frac{\lambda \lambda}{(\lambda \quad \lambda)^2}. \tag{A.3}$$

r m r 4 r m o 4 r - g o r m om o r . T k g o 44o , o r 4 g r r

$$+ \frac{\lambda \lambda_{1}}{(\lambda \quad \lambda_{1})^2} \sum_{+1}^{2} \frac{\lambda \lambda}{(\lambda \quad \lambda)^2}, \tag{A.4}$$

$$+ \frac{\lambda \lambda_{1}}{(\lambda \quad \lambda_{1})^2} \sum_{+ \; 2} \frac{\lambda \lambda}{(\lambda \quad \lambda)^2}. \tag{A.5}$$

rg o r of (A.4) (A.5) r 4o r g r - g o r m mm o . r 4 r, o ro m o r o o r g g 0 , , (A m o 2.2 4o rg 4 , 4 o , r + (1/)), m o 4 m

$$\frac{\lambda \lambda_{1}}{(\lambda \quad \lambda_{1})^2} \qquad \frac{\lambda^2}{(\ )^2}. \tag{A.9}$$

o r or o o mm o , ro m g m g , 4 r $\rho(\lambda)$ of orm m r 4 4o g m r of g . or r 4 , 4o r 4 of - mm r 4 4o r 4 m r 4 , g g λ, for $1, \ldots,$. for 4 m r m r 4 4 r

$$\rho(\lambda) + \ ^{1} \sum \delta(\lambda), \tag{A.10}$$

r $\delta(\lambda)$ 4 fi 4 o $\lambda \in \mathbb{R}$. m 4o r 4 m r 4 r r from m 4 4 $\rho(\lambda)$ k 4o rg , m g

$$\int \rho(\lambda)(\lambda)\ \lambda \qquad \int \rho(\lambda)(\lambda)\ \lambda \tag{A.11}$$

for 4o o fi 4 o (λ). m $\rho(\lambda)$ 4o , , 4om 4 or (o (ρ)) ff r o (ρ). or o o 4o 4 , m $(\rho) + (\alpha, \beta)$ for om $\alpha, \beta \in \mathbb{R}$, o g o r 4 m of gr o (A.11) (α, β). o r, r 4 o o of 4 r . g r r g mm o (A.4) (A.5) gr o of fi 4 o

$$\lambda(\lambda) + \frac{\lambda \lambda}{(\lambda \quad \lambda)^2} \tag{A.12}$$

ro m r $\rho(\lambda)$ g (A.10),

$$\frac{1}{\ } \sum_{+1}^{2} \frac{\lambda \lambda}{(\lambda \quad \lambda)^2} + \int_{\alpha}^{\lambda_{1}} \rho(\lambda)_{\lambda}(\lambda)\ \lambda, \tag{A.13}$$

$$\frac{1}{\ } \sum_{+\ 2\lambda}$$

o r o r g of (A.15) 4 ε. g o $\rho(\lambda)$ (λ) r ff r for λ r (ρ) ‖ λ, m m gr o r, r g m r or om or r , o o

$$\int_{\alpha}^{\lambda \ \ \varepsilon} (\lambda)\rho(\lambda) \ \lambda \ + \ \lambda \frac{(\lambda \quad \varepsilon)\rho(\lambda \quad \varepsilon)}{\varepsilon} \quad \lambda \int_{\alpha}^{\lambda \ \ \varepsilon} \frac{\rho(\lambda) \quad \lambda\rho \ (\lambda)}{\lambda \quad \lambda} \ \lambda. \tag{A.16}$$

r m r g of (A.16) ε 0 m o 4 m

$$\lambda \frac{(\lambda \quad \varepsilon)\rho(\lambda \quad \varepsilon)}{\varepsilon} \qquad \frac{\lambda^2 \rho(\lambda)}{\varepsilon}. \tag{A.17}$$

4o m o r g of (A.16) o

$$\left| \lambda \int_{\alpha}^{\lambda \ \ \varepsilon} \frac{[\rho(\lambda) \quad \lambda\rho \ (\lambda)]}{\lambda \quad \lambda} \ \lambda \right| \leqslant \lambda \left[\underset{\lambda \ (\alpha,\lambda \ \ \varepsilon]}{} \rho(\lambda) \quad \lambda\rho \ (\lambda) \right] \int_{\alpha}^{\lambda \ \ \varepsilon} \frac{1}{\lambda \quad \lambda} \ \lambda$$

$$+ \lambda \left[\underset{\lambda \ (\alpha,\lambda \ \ \varepsilon]}{} \rho(\lambda) \quad \lambda\rho \ (\lambda) \right] \left(\frac{\lambda \quad \alpha}{\varepsilon} \right). \tag{A.18}$$

fo o 4o m r g of (A.16) 4 g ((1/ε)) om m ε 0 r m, 4 (1/ε). 4om (A.17) (A.18) o o ε 0 m o 4 m

$$\int_{\alpha}^{\lambda \ \ \varepsilon} (\lambda)\rho(\lambda) \ \lambda \qquad \frac{\lambda^2 \rho(\lambda)}{\varepsilon}. \tag{A.19}$$

o 4 r $\rho \ (\lambda)$ o , r g for r o r gr o f of (A.18) o o om : o 4o g λ, r $\rho \ (\lambda)$ o 4o om g r m λ ε. r gr 4o rg o ro o (A.11) r 4o o g (A.18), m g gr m (A.16) ((1/ε)), ro $\rho(\lambda)$ ff r m g o r oo 4o g λ. m g o r for r g of (A.13) 4o r g fo o g :

$$\int_{\alpha}^{\lambda \ \ 1} (\lambda)\rho(\lambda) \ \lambda + \int_{\alpha}^{\lambda} (\lambda)\rho(\lambda) \ \lambda \quad \int_{\alpha}^{\lambda} (\lambda) \left[\rho(\lambda) \quad \rho(\lambda) \right] \ \lambda$$

mm r , 4om (A.21), (A.22) (A.17

To o m for r o of , ( ), ff r (B.6) r 4 o o o

$$( ) + \frac{\partial}{\partial} \int_{0( )} \int_{(,)} (,) \quad \text{(B.7)}$$

$$+ \frac{\partial^{0}}{\partial}( ) \int_{(, ^{0}( ))} (, ^{0}( )) \int_{0( )} \frac{\partial}{\partial} \left[\int_{(,)} (,) \right]$$

$$+ \int_{0( )} ((,),) \frac{\partial (,)}{\partial} .$$

o o r o , r m 4o 4 (,) , m $^{0}( )$ (,) o .

C. Derivation of main result 3

r 44or g o ( ), g (B.7), r 4 o m o 4 r o for ( ) m , . m g (2.2), o (,) rg /or r m , rg m o 4 4o r o 4o r o of m ro or o o $_{2}$ ^{2},

$$\frac{\lambda^2}{( )^2} \quad \frac{\lambda^2}{( )^2}. \quad (.1)$$

4 ,

$$^{0}( ) + \frac{\lambda}{\ } =, \quad (.2)$$

$$(,) + \frac{\lambda}{[( )^2 \ \lambda^2]^{1/2}}, \quad (.3)$$

$$\frac{\partial}{\partial} \left((,) \right) + \frac{\lambda( )^3}{}$$

o , 4 g for gr $_1(\cdot)$, $_2(\cdot)$ $_3(\cdot)$. B g g (.4), r o for r g (0, $_1(\cdot)$]

$$(\lambda^2)^{5/2}(^{1/2} \lambda) \leqslant (_1 \lambda^2)^{5/2}(^{1/2}_1 \lambda). \qquad (.7)$$

fo o

$$\left(1 \frac{\lambda^2}{}\right)$$

fo o

$$\left(^{/2}\right.$$

r 4o > 0 $\lambda^2/$ > 0 o

$$\varphi( ) + \frac{[3, \rho(\lambda)]^2}{4\pi}(\, , \lambda^2)\left[1 , (\lambda^2/) , (\lambda^2/)^{1/2}\right]$$

$$\geqslant \frac{[3, \rho(\lambda)]^2}{4\pi} \, . \qquad (\ldots1)$$

4 g of r $+ \frac{[3, \rho(\lambda)]^2}{4\pi}$ o o

$$_3( ) \leqslant 8 \left(\frac{[3, \rho(\lambda)]^2}{4\pi}\right)^{3/2} \int_{\frac{[3, \rho(\lambda)]^2}{4\pi} \; _2( )/}^{\prime} {}^{1/2} \, . \qquad (\ldots18)$$

rom (.3), o r m of gr o 4o rg $\frac{[3, \rho(\lambda)]^2}{4\pi}$ $_2( )/$ 1 gr (.18) 4o rg o 4o . r for , $_3( )$ o m 4 g $^{3/2}$. o o $_2( )$ 4 g $(^{3/2})$ (o o o o r m , 4 o r m 4 $^{3/2}$). o 4 of r o of $_1$ $_2$, g $\varphi( )$ r 4 m m m $+ \lambda^2$ ($_1$, $_2$),

$$\varphi(\lambda^2)/ \leqslant \varphi( )/ \leqslant 1 \qquad (\ldots19)$$

for ($_1$, $_2$). g o (.6), $_2( )$

$$( ) \;\; ^1 \leqslant \; _2( ) \leqslant \; ( ) \;\; ^{\varphi(\lambda^2)/} , \qquad (\ldots20)$$

r .

$$( ) \int_{_1( )}^{_2( )} \left(1 , \frac{\lambda^2}{}\right)^{5/2} (\; ^{1/2} , \lambda) \; . \qquad (\ldots21)$$

g m o 4 ro m o for $_1( )$ $_2( )$ g (.2) (.3), gr (.21) g of r m 4 (g r [( ), , ,1]' 4om) o o m o 4 r,

$$( ) \; \frac{2^4 \pi^{3/2}}{3^4 [, \rho(\lambda)]^3} \; ^{3/2}. \qquad (\ldots22)$$

r mor , 4om $\varphi(\lambda^2)/$ 0 (.20) o o m o 4

$$( ) \;\; ^1 \leqslant \; _2( ) \leqslant \; ( ). \qquad (\ldots23)$$

4om (.23) (.15) (.18) o o rg 4 g ( ) + $\left(\frac{^{3/2}}{,\, ^3}\right)$.