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Cluster synchrony in systems of coupled phase oscillators with higher-order coupling

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We dy he henomenon of cl e ynch ony ha occ in en emble of co led ha e o cilla o hen highe -o de mode domina e he co ling be een o cilla o . Fo he ime, e de elo a com le e analy ic de c i ion of he dy namic in he limi of a la ge n mbe of o cilla o and e i o an ify he deg ee of cl e ynch ony, cl e a y mme y, and i ching. We e a ia ion of he ecen dimen ionali y - ed c ion echni e of O and An on en [Chao **18**, 037113 (2008)] and nd an analy ic de c i ion of he deg ee of cl e ynch ony alid on a globally a ac ing manifold. Sha ed by hi manifold, he e i an in ni e family of eady - a e di ib ion of o cilla o , e l ing in a high deg ee of m l i abili y in he cl e a y mme y. We al o ho ho h o gh e e nal fo cing he deg ee of a y mme y can be con olled, and gge ha y em di lay ing cl e ynch ony can be ed o encode and o e da a.

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I. INTRODUCTION

La ge y em of co led o cilla o occ in many e -

Cl e y nch ony ha been died in many con e , fo
e am le, in ne o k of ha e o cilla o i h [

i h ha e θ and na al f e enc y ω a ime t. Since o cilla o a e con e ed f m a i f y he con in i y e a ion $\partial_t f + \partial_\theta(f \theta) = 0$, gi ing

$$\partial_t f + \partial_\theta \left\{ f \left[\omega + \frac{K}{2i}(r_2 e^{-2i\theta} - r_2^* e^{2i\theta}) \right] \right\} = 0. \quad (6)$$

To anal y e E . (6), e nd i con enien o de ne he y mme ic and an i y mme ic a of f, f_s, and f_a, a

$$f_{s/a}(\theta,\omega,t) = [f(\theta,\omega,t) \pm f(\theta + \pi,\omega,t)]/2, \quad (7)$$

he e f_s and f_a a e y mme ic and an i y mme ic i h e ec o an la ion b y π, e ec i el y , in he en e ha $f_s(\theta + \pi,\omega,t) = f_s(\theta,\omega,t)$ and $f_a(\theta + \pi,\omega,t) = -f_a(\theta,\omega,t)$. We no e ha f i a ol ion of E . (6) if $f = f_s + f_a$ and f_s and f_a a e bo h ol ion of E . (6). Th , e can d y e a a el y he y mme ic and an i y mme ic d y namic of ol ion f.

A. Symmetric dynamics

hile he an i y mme ic den i y f_a emain he ame. Th ,
he onl y change in $|r_1$

P oblem ha emain o en incl de gene ali a ion ch a he e ence of noi e and co ling f nc ion i h o o mo e ha monic . Th fa he o k of O and An on en [19

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