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Emergence of synchronization in complex networks of interacting dynamical systems

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Abstract

We study the emergence of coherence in large complex networks of interacting heterogeneous dynamical systems. We show that for a large class of dynamical systems and network topologies there is a critical coupling strength at which the systems undergo a transition from incoherent

in [17] is

$$1 = kZ(\eta), \tag{6}$$

where

$$Z(\eta) = \lim_{n \to \infty} \left[\sum_{p=0}^{n} \frac{q'(x_{p+1})\delta_{n+1} g(x_p)\eta^{p-n-1}}{\delta_{p+1}} \right]. \tag{7}$$

For large $n, \delta_{n+1}/\delta_{p+1}$ grows with n on average as $\chi(\mu, x_0)^n$

matrices with mixed positive/negative entries, we have found that if the average of the nonzero elements of the matrix is large enough, the eigenvalue with largest magnitude is also real and it is well separated from the eigenvalue with the next largest magnitude. For details, see the Appendix of Ref. [8].

3. Networks of coupled dynamical systems

In the previous section we have reviewed results in cases

Using $M(t, y(0))M^{-1}(\tau, y(0)) = M(t - \tau, y(\tau))$ and defining $T \equiv t - \tau$, we get

$$\delta y^{(j)}(t) = \delta y^{(j)}(0) + \int_0^t M(T, y^{(j)}(t - T))K(y^{(j)}(t - T)) \times$$

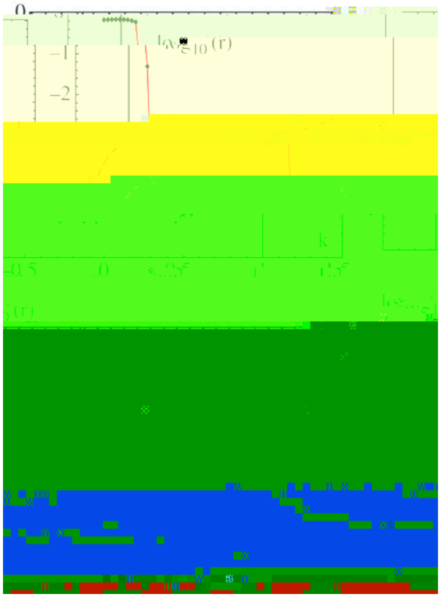

Fig. 1. Logarithm of the order parameter, $\log_{10} r$, as a function of the coupling strength k for example 1 (identical noiseless maps), for a scale-free network with degree distribution $P(d) \propto d^{-3}$ if $d \geq 100$, and 0 otherwise, for (a) $N = 10$

Fig. 2. Order parameter as a function of the coupling strength for example 2 (heterogeneous noisy maps), for a scale-free network with in- and out-degree

Fig. 5. Order parameter as a function of the coupling strength for the heterogeneous ensemble of noiseless chaotic Lorenz oscillators, for a scale-free network with in- and out-degree distribution P