

important, the classical diffraction stack always provides a potential from its projections, i.e. from seismic data. The problem of

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are surfaces in model space that come from fixing a point $\mathbf{d} = (r, s, t)$ in the data and finding the surface of image points

corresponding pair of projection operators (Miller, 1983):

associated with \mathbf{d} by the equation $f(\mathbf{m}) = \mathbf{d}$. (2) Given an object function $f(\mathbf{m})$ defined on the model

surface by L , it is the set

space we obtain a data function $f(\mathbf{d})$ defined at each

medium, so that

$$\nabla^2 G_0(\mathbf{x}, \mathbf{y}, \omega) + \frac{\omega^2}{c_0^2(\mathbf{x})} G_0(\mathbf{x}, \mathbf{y}, \omega) = -\delta(\mathbf{x} - \mathbf{y}). \quad (6)$$

With these definitions, equation (4) can be recast as an integral equation, analogous to the Lippman-Schwinger equation of quantum mechanics (see, e.g., Taylor, 1972),

$$u(\mathbf{y}, \mathbf{s}, \omega) = G_0(\mathbf{y}, \mathbf{s}, \omega) + \omega^2 \int d^3\mathbf{x} G_0(\mathbf{y}, \mathbf{x}, \omega) f(\mathbf{x}) u(\mathbf{x}, \mathbf{s}, \omega).$$

When evaluated at receiver position \mathbf{r} , this equation gives the observed total field as a sum of the incident field within the background model G_0 plus the scattered field, represented by the integral term. Denoting the scattered field by

$$\left[\nabla_{\mathbf{x}} \tau(\mathbf{x}, \mathbf{y}) \right]^2 = c_0^{-2}(\mathbf{x}), \quad (9)$$

and the amplitude or geometrical spreading term A satisfies the transport equation

$$A(\mathbf{x}, \mathbf{y}) \nabla_{\mathbf{x}} \tau(\mathbf{x}, \mathbf{y}) \nabla_{\mathbf{x}} A(\mathbf{x}, \mathbf{y}) \cdot \nabla_{\mathbf{x}}(\mathbf{x}, \mathbf{y}) = 0, \quad (10)$$

along the ray connecting the points \mathbf{x} and \mathbf{y} .

Substituting for G_0 in equation (8) gives

$$\begin{aligned} u_{sc}(\mathbf{r}, \mathbf{s}, \omega) &= \omega^2 \int d^3\mathbf{x} A(\mathbf{r}, \mathbf{x}) A(\mathbf{x}, \mathbf{s}) \\ &\times \exp \left\{ i\omega \left[\tau(\mathbf{r}, \mathbf{x}) + \tau(\mathbf{x}, \mathbf{s}) \right] \right\} f(\mathbf{x}) \\ &= \omega^2 \int d^3\mathbf{x} A(\mathbf{r}, \mathbf{x}, \mathbf{s}) \exp \left[i\omega \tau(\mathbf{r}, \mathbf{x}, \mathbf{s}) \right] f(\mathbf{x}). \end{aligned} \quad (11)$$

will contain a convolution with the source wavelet. One can then shift the time derivative onto the source wavelet itself (see Tarantola, 1984). For reasons explained in a later section, we keep these equations in the given form. In the final section, we briefly discuss the effect of a band-limited source.

Analogy with the classical Radon transform

As r , s , and t vary over the data, the acoustic GRT gives weighted integrals of the scattering potential over isochron surfaces in the model. We derive an approximate inverse

formula (Radon, 1917; Gelfand et al., 1966; Deans, 1983)

$$\begin{aligned}
 f(\mathbf{x}_0) &= -\frac{1}{8\pi^2} \int d^2\xi \left[\frac{\partial^2}{\partial p^2} f^\Delta(\xi, p) \right]_{p=\xi \cdot \mathbf{x}_0} \\
 &= -\frac{1}{4\pi} \int d^2\xi \frac{\partial^2}{\partial p^2} f^\Delta(\xi, p = \xi \cdot \mathbf{x}_0). \quad (16)
 \end{aligned}$$

equation (15), we can write for the filtered transform

$$\begin{aligned}
 \frac{\partial^2}{\partial p^2} f^\Delta(\xi, p) &= \frac{\partial^2}{\partial p^2} \int d^3\mathbf{x} \delta(p - \xi \cdot \mathbf{x}) f(\mathbf{x}) \\
 &= \int d^3\mathbf{x} \delta''(p - \xi \cdot \mathbf{x}) f(\mathbf{x}).
 \end{aligned}$$

Equation (16) is the 3-D version of the filtered backprojection algorithm of X-ray tomography (see, e.g., Herman, 1980). For fixed ξ , the function $f^\Delta(\xi, p)$ is a one-dimensional function of p

Combining this with equation (16) gives

$$f(\mathbf{x}_0) = -\frac{1}{4\pi} \int d^2\xi \int d^3\mathbf{x} \delta''[\xi \cdot (\mathbf{x}_0 - \mathbf{x})] f(\mathbf{x}). \quad (17)$$

ing gives directly what may be called a "filtered" generalized Radon transform in which the integrals are taken over surfaces

Radon transform. To obtain the identification, first shift the origin of coordinates for the integrals to the point x_0 . Letting

Consider first $\nabla_{\mathbf{x}} \tau(\mathbf{r}, \mathbf{x}_0)$. Since rays are perpendicular to surfaces of equal traveltime (phase) and traveltime increases as \mathbf{x}_0 moves away from \mathbf{r} , this gradient points in the opposite direction from the ray that leaves \mathbf{x}_0 and reaches \mathbf{r} in the background model, or along the ray that arrives at \mathbf{x}_0 from \mathbf{r} .

gives the weighting function dW directly,

$$dW(\mathbf{r}, \mathbf{x}_0, \mathbf{s}) = \frac{1}{\pi^2} d^2\xi(\mathbf{r}, \mathbf{x}_0, \mathbf{s}) \frac{|\cos^3 \alpha(\mathbf{r}, \mathbf{x}_0, \mathbf{s})|}{c_0^3(\mathbf{x}_0)A(\mathbf{r}, \mathbf{x}_0, \mathbf{s})}, \quad (26)$$

and the final inversion formula

arrives at \mathbf{x}_0 from the source \mathbf{s} . The geometry is illustrated in Figure 6. We call these gradient vectors the incident and scattered rays at the image point \mathbf{x}_0 .

From the eikonal equation (9), the magnitudes of the incident and scattered rays are equal to $1/c_0(\mathbf{x}_0)$, the slowness of the background model at the point \mathbf{x}_0 . The total traveltime

$$\langle f(\mathbf{x}_0) \rangle = \frac{1}{\pi^2} \int d^2\xi(\mathbf{r}, \mathbf{x}_0, \mathbf{s}) \times \frac{|\cos^3 \alpha(\mathbf{r}, \mathbf{x}_0, \mathbf{s})|}{c_0^3(\mathbf{x}_0)A(\mathbf{r}, \mathbf{x}_0, \mathbf{s})} u_{sc}(\mathbf{r}, \mathbf{s}, t = \tau_0). \quad (27)$$

The inversion integral we have derived is given explicitly in

The analysis described above carries through with only minor changes to yield the 2-D acoustic inversion formula

$$\langle f(\mathbf{x}_0) \rangle = \frac{1}{\pi} \int d\xi(r, \mathbf{x}_0, \mathbf{s})$$

er here zero-offset and fixed-offset experiments in a constant background velocity. The zero-offset migration formula was first derived in Norton and Linzer (1981) by a different approach; the fixed-offset formula was derived in Beylkin (1985).

Zero-offset migration

factor $A(\mathbf{r}, \mathbf{x}, \mathbf{r}) = (4\pi|\mathbf{x} - \mathbf{r}|)^{-2}$ and substituting into equation (27) yields

$$\begin{aligned} \langle f(\mathbf{x}) \rangle &= \frac{16}{c_0^3} \int_{r_3=0} d^2\mathbf{r} \frac{x_3}{|\mathbf{x} - \mathbf{r}|} u_{sc}(\mathbf{r}, t = 2|\mathbf{x} - \mathbf{r}|/c_0) \\ &= \frac{16}{c_0^3} \int d^2\mathbf{r} \cos \theta u_{sc}(\mathbf{r}, t = 2|\mathbf{x} - \mathbf{r}|/c_0). \end{aligned} \quad (29)$$

ever it is possible to specify ξ explicitly in terms of the Cartesian coordinates of the experiment.

Fixed-offset experiments

Consider next a fixed-offset experiment on the surface of a half-space with constant background velocity. The experiment

A second algebraic derivation follows directly from equation (28). By definition, the surface integral

source-receiver pair. Let $\mathbf{h} = (h_1, h_2, 0)$ be the half-offset vector, so that the receiver position $\mathbf{r} = \mathbf{m} + \mathbf{h}$ and the source

where the magnitude of the vector cross-product is the Jacobian factor. This again yields equation (29); moreover, if the

$$\xi = \left[2 + (\mathbf{x} - \mathbf{s}) \cdot (\mathbf{x} - \mathbf{r}) / |\mathbf{x} - \mathbf{s}| |\mathbf{x} - \mathbf{r}| \right]^{1/2}$$

(a)

(b)

(m)

(m)



A change of variables can now be made from ξ to m using equations (30) and (31); however, the algebra is dense. For simplicity, consider the 2-D case and work with the angular variables defined in Figure 7; the polar angle θ determines

in the scattering potential. Consider the velocity function defined by the relation

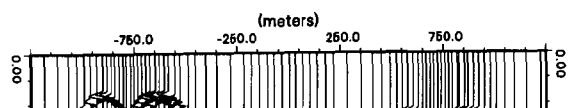
$$c^{-2}(z) = 1 + \Theta(z - z_0),$$

source to the image point x , and α_r is the angle between the vertical and the ray from the receiver to the image point x . Then,

$$\begin{aligned} \theta &= \frac{1}{2}(\alpha_s + \alpha_r) + \pi \\ &= \frac{1}{2} \left(\tan^{-1} \frac{m-h}{z} + \tan^{-1} \frac{m+h}{z} \right) + \pi, \end{aligned} \quad (32)$$

depth of the reflector. Taking $c_0^{-2} = 1$ gives the scattering potential

$$f(z) = \Theta(z - z_0).$$



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coverage of tangent planes passing through an image point;

faces, by setting the scattering potential to be

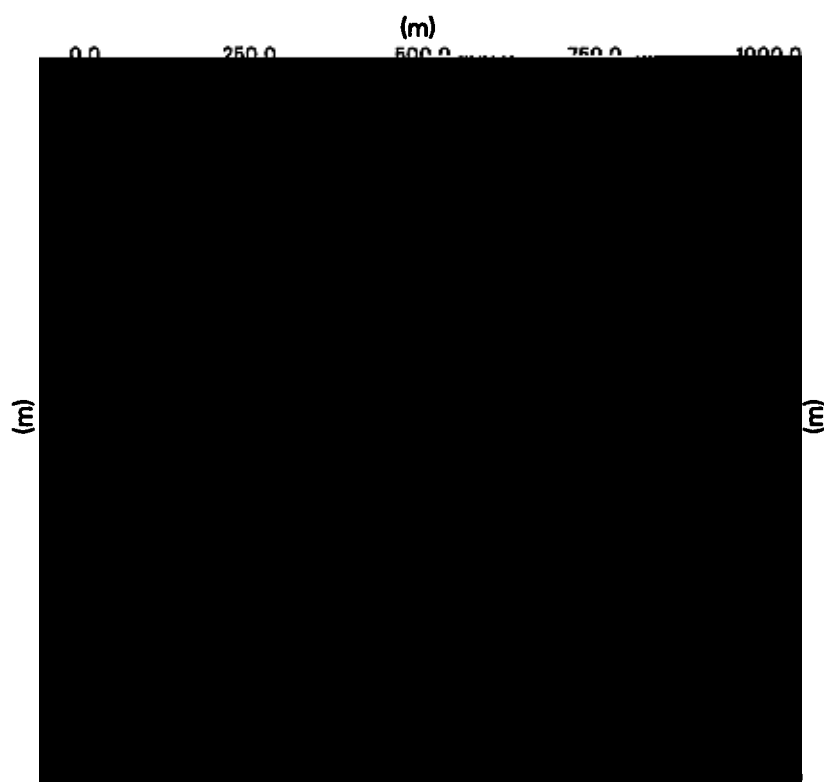
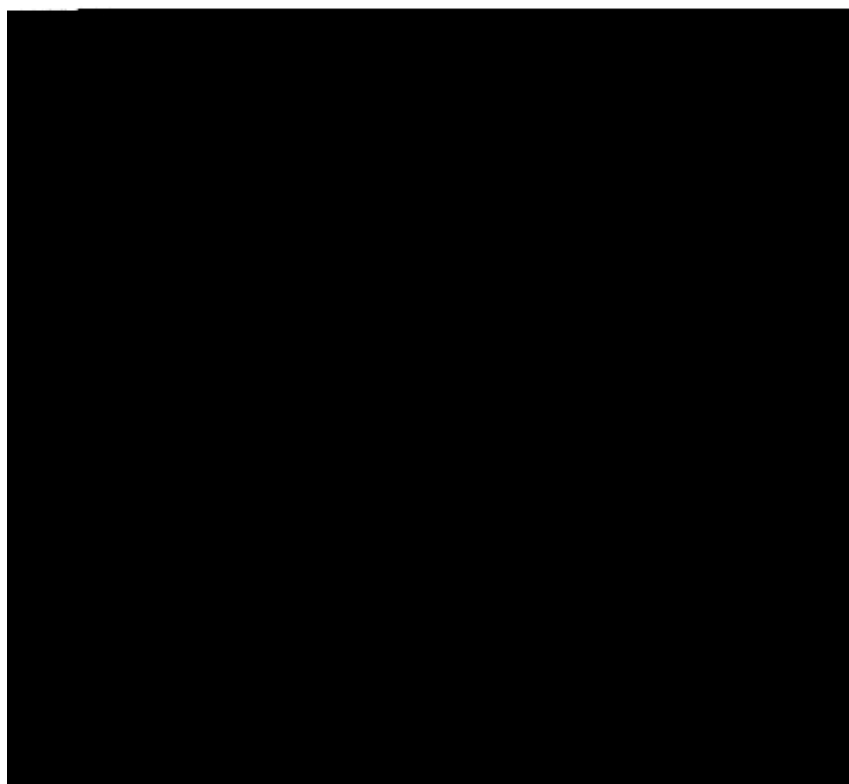


FIG. 15. Reconstruction from the synthetic DVSP experiment. (a) Ten wiggle plots showing profiles of the reconstructed scattering potential are superimposed on a gray level display of the same image. Vertical profiles of the two

The first example consists of data for a single source — 2-D acoustic finite difference program, 160 times synthetic

multiple-receiver experiment with the source and receivers arranged as in Figure 1. The scattering object consisted of a family of point scatterers, which were separated by roughly one wavelength at the central frequency of the source and

scattered data were obtained for 80 source-receiver pairs. The source wavelet was a Blackman-Harris window with a duration of 21.3 ms (which contains frequencies ranging from 0 to about 50 Hz).

mining the shapes and locations of the surfaces, the method is easily adapted to general source-receiver geometries and velocity models. Moreover, the dependence of spatial resolution on the geometry of the experiment, the reconstruction algorithm, and the assumptions about the medium is explicit in

perimental geometry and the finite bandwidth of the source wavelet.

The first issue is the relation between the available source-receiver pairs and the spatial dip spectrum of the reconstructed object. Locally, a restriction on the number of source

performing just surface experiments, just borehole experiments, or both. It is also possible to describe an ideal experiment for a given configuration

surfaces in the generalized Radon transform, and hence, the set of tangent planes (parameterized by ξ) available at each image point. Recall from the discussion of the original Radon

the level set of τ is normal to the corresponding isochron together with the family of isochrons corresponding to data

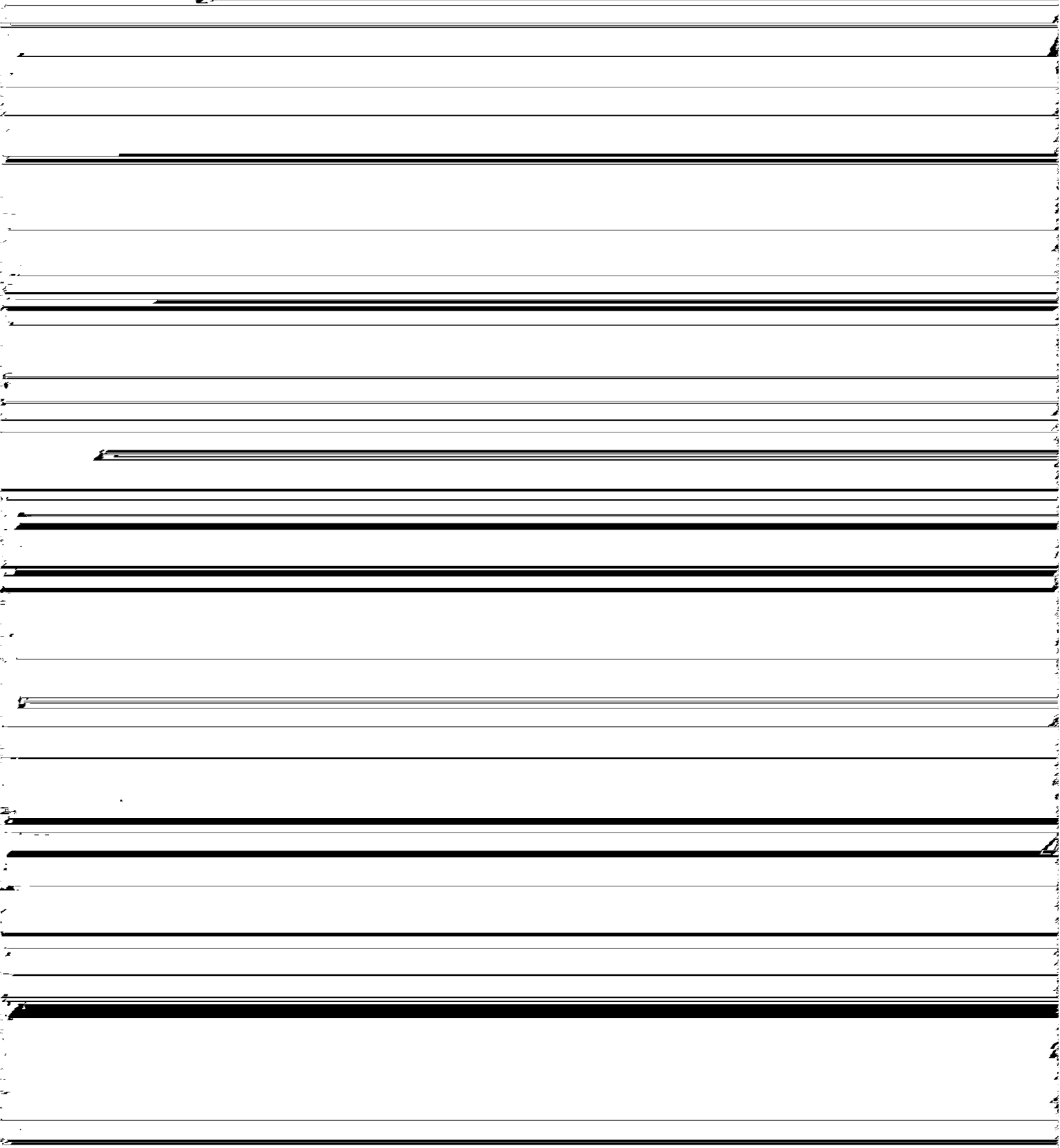
where surface is Eq. (17) and equation (23)

points one-half wavelength from the associated reflection-time

$$\Delta p = \frac{c_0(x_0)}{2 \cos \alpha} \Delta t. \quad (36)$$

surface. This last figure illustrates the obliquity effect directly in the image domain.

The mapping of the data into the spatial Fourier spectrum



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Miller, D. E. and Dooel, J., 1986, Reef delineation by multiple offset

by its Fourier integral yielding

integrals which express these derivatives as det. For large ϵ