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Three-dimensional inverse scattering for the wave equation with variable speed: near-field formulae using point sources

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Abstract. We consider the inverse scattering problem for the wave equation with variable speed where the region of interest is probed with waves emanating from point sources. We obtain a three-dimensional trace-type formula, which gives the unknown speed in terms of data and the interior wavefield.

1. Introduction

Here x and y are points in $\boldsymbol{R}^3$, k is a real scalar, and δ is the three-dimensional delta function. The index of refraction $n(x)$ we assume to be a positive, bounded, real-valued function which is identically one outside some bounded region Ω.

We are interested in particular solutions of (2.1) which we specify with the help of the functions

[redacted]

which satisfy

$$(\nabla^2 + k^2)G_0^{\pm}(k, x-y) = \delta(x-y). \qquad (2.2\pm)$$

We now specify solutions G^+ and G^- of (2.1) as solutions of the integral equations

$$G^{\pm}(k, x, y) = G_0^{\pm}(k, x-y) + \int_{\Omega} G_0^{\pm}(k, x-z)k^2V(z)G^{\pm}(k, z, y)\,\mathrm{d}z \qquad (2.3\pm)$$

where $V = 1 - n^2$.

There are two techniques for showing that (2.3+) and (2.3−) each have unique solutions. One technique [5] shows that for almost every k, (2.3) has a unique solution with $G|V|^{1/2}$ in L^2. Another technique [6], which uses the limiting absorption principle, shows that for every k, (2.3) has a unique solution in a certain weighted Sobolev space. Both these techniques apply in the present case when V is bounded and has compact support.

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Proof. The proof, based on the use of Green's formula, is similar to the correspond-

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$$\int_{\partial\Omega}\left(G_0^-(z-x)\frac{\partial}{\partial\nu}G_0^+(z-y)-G_0^+(z-y)\frac{\partial}{\partial\nu}G_0^-(z-x)\right)\mathrm{d}S_z$$
$$=G_0^-(y-x)-G_0^+(x-y). \tag{2.6}$$

Proof. We set $n(x)$ equal to one in theorem 1.

Corollary 2. Suppose the hypotheses of theorem 1 hold and $G^\pm = G_0^\pm + G_{sc}^\pm$. Then

$$G_{sc}^-(y,x)-G_{sc}^+(x,y)=\int_{\partial\Omega}\left(G_0^-(z-x)\frac{\partial}{\partial\nu}G_{sc}^+(z,y)-G_{sc}^+(z,y)\frac{\partial}{\partial\nu}G_0^-(z-x)\right.$$

[redacted]

Proof. We substitute $G^+ = G^+ + G^+$ into (2.1), simplify, and evaluate at $k = 0$.

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