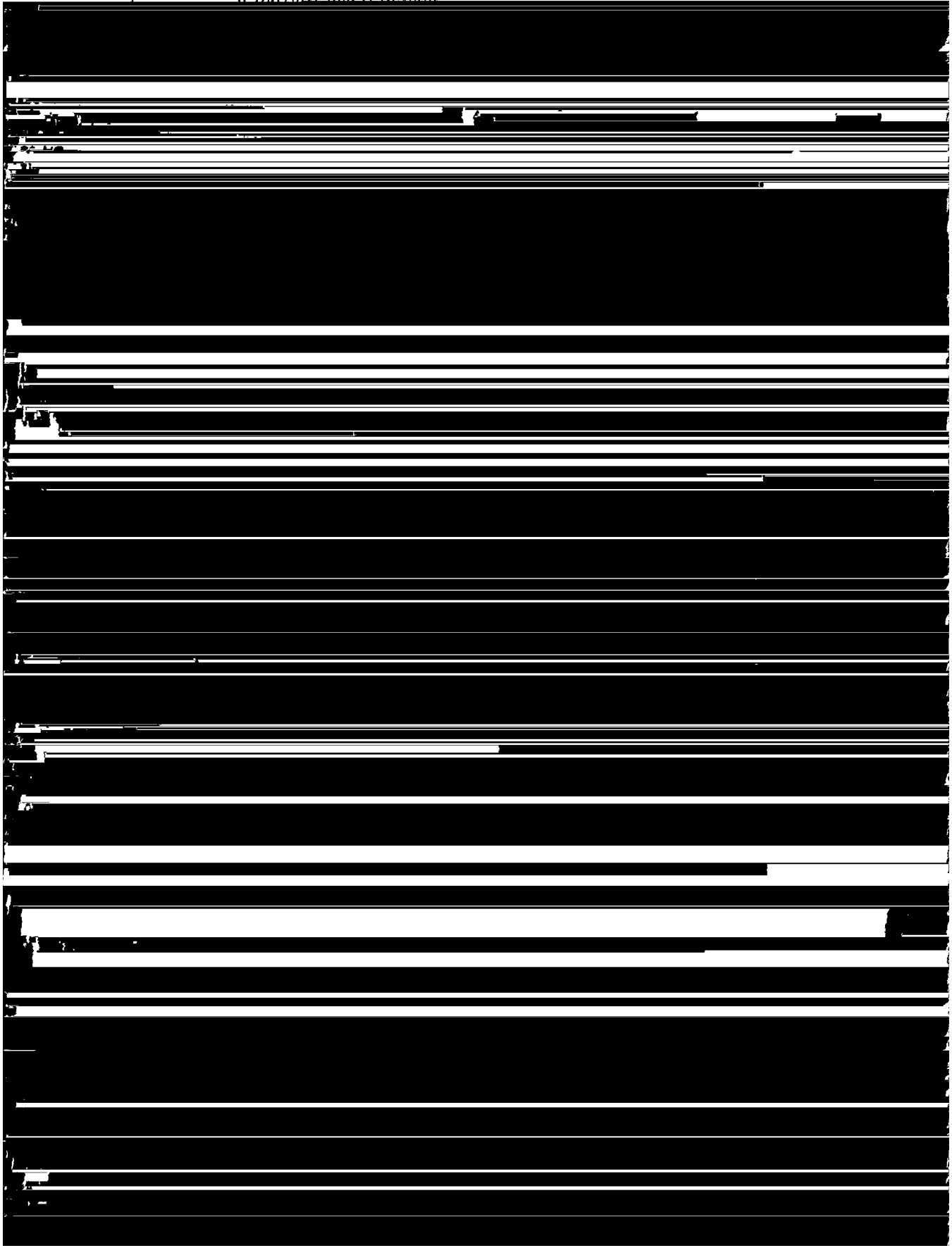# On double integrals over spheres

Radon transform. In § 4 using results of § 3 we derive some formulae in inverse scattering theory extending results of Devaney (1982a). We then discuss diffraction tomography where the inversion formula of § 3 yields a generalisation to what can be called multifrequency diffraction tomography. The same inversion formula is also used in the derivation of migration algorithms used in inverting seismic prospecting data (Beylkin and Burridge 1987a, b), thus establishing a relationship between multifrequency diffraction

where ξ and η are defined as in the lemma. Using (3.10) in (3.8) and setting $r = \rho(\lambda + \mu)$ we have

$$I = \frac{\lambda\mu}{\Omega_{n-1}(\lambda+\mu)} \frac{1}{C(1-\gamma)} \int_0^\infty |p|^{n-1} \, \mathrm{d}|p|$$

$$\times \int_{S^{n-1} \times S^{n-1}} f\left(|p| \frac{(\lambda\xi+\mu\eta)}{r}\right) \frac{W(r/(\lambda+\mu))}{r \sin^{n-3}\theta} \, \mathrm{d}\xi \, \mathrm{d}\eta \qquad (3.11)$$

where

$$r = |\lambda\xi + \mu\eta|. \qquad (3.12)$$

Next set

$$|p| = rk \qquad \mathrm{d}|p| = r \, \mathrm{d}k \qquad (3.13)$$

to get

$$I = \frac{\lambda\mu}{2\Omega_{n-1}(\lambda+\mu)} \frac{1}{C(1-\gamma)} \int_{-\infty}^\infty |k|^{n-1} \, \mathrm{d}k$$

$$\times \int_{S^{n-1} \times S^{n-1}} f(k(\lambda\xi+\mu\eta)) \frac{r^{n-1} W(r/(\lambda+\mu))}{\sin^{n-3}\theta} \, \mathrm{d}\xi \, \mathrm{d}\eta \qquad (3.14)$$

where

$$b(y, \xi, \eta) = \frac{|\lambda\xi + \mu\eta|^{n-1}}{\cdots} \frac{2\lambda\mu}{\cdots} \frac{1}{\cdots} W\left(\frac{|\lambda\xi + \mu\eta|}{\cdots}, y\right) \cdots \quad (3.19)$$

Using (2.11), (3.15), (3.19) and integrating over ψ, we have from (3.25)

$$g_B(y) = \frac{1}{(2\pi)^n} \int_{k_{min}}^{k_{max}} k^{n-1} \, \mathrm{d}k \, \frac{1}{1 - \gamma(y)} \int_{\gamma(y)}^{1} \mathrm{d}\rho \int_{S^{n-1}} \mathrm{d}\nu$$

$$\times W(\rho, y)\rho^n(\lambda + \mu)^n \hat{g}(k\rho(\lambda + \mu)\nu) \exp[ik\rho(\lambda + \mu)\nu \cdot y]. \tag{3.26}$$

Changing the order of integration in (3.26) we arrive at

$$g_B(y) = \frac{1}{1 - \gamma(y)} \int_{\gamma(y)}^{1} \mathrm{d}\rho W(\rho, y) g_\rho(y) \tag{3.27}$$

where

$$g_\rho(y) = \frac{1}{(2\pi)^n} \int_{\rho(\lambda + \mu)k_{min} \leqslant |p| \leqslant \rho(\lambda + \mu)k_{max}} \mathrm{d}p \, \hat{g}(p) \exp(ip \cdot y). \tag{3.28}$$

Formula (3.27) can be interpreted as a superposition of band-limited reconstructions g_ρ.

Given the definitions of the operators R^* and K we substitute (3.30) into (3.18) to obtain

$$g(y) = (R^*Kv)(y) \qquad\qquad (3.34)$$

The validity of the Born approximation in the context of the quantum-mechanical scattering can be justified under the assumption of smallness of the potential or for high energies (frequencies) for a given potential.

Our inversion formula provides a direct reconstruction of the potential and the inter-atomic distance function for the linearised inverse problem. Indeed, applying theorem 1 specialised to dimension $n = 3$ (see also § 3, remark 1) with $g \equiv V$ and using (4.4) we have