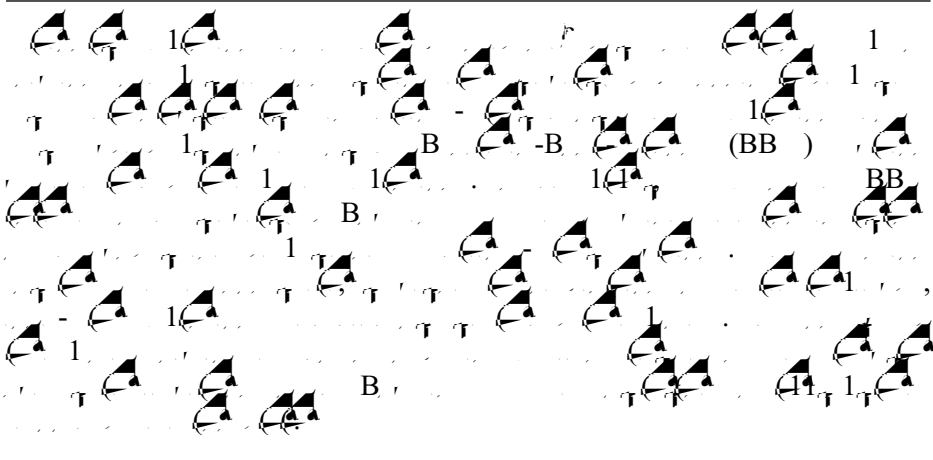
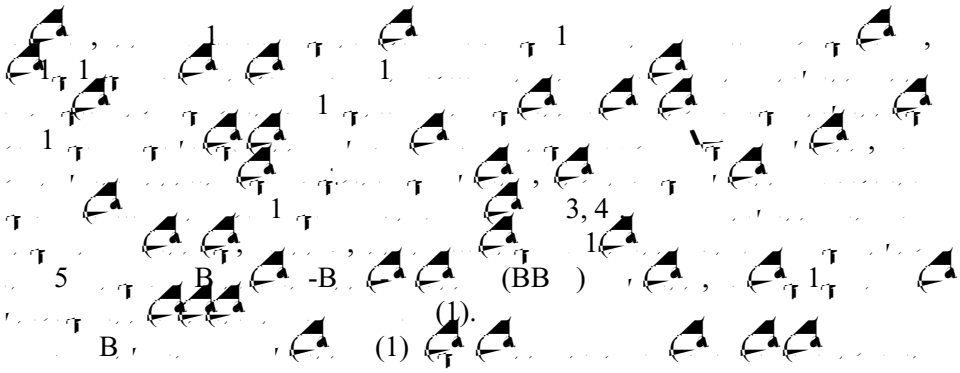


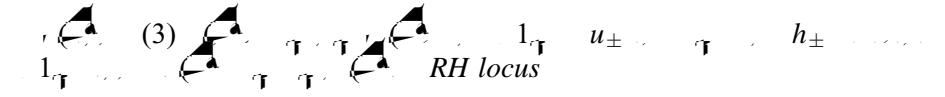
# Spin Exchange for a Regulated Bull Market

By Gennady A. El, Mark A. Hoefler , and Michael Shearer



## 1. Introduction





$$u_{\pm} = h_{\mp} \left( \frac{2}{h} \right)^{k/2} \quad (4)$$



## 2. Expansion shock Riemann data



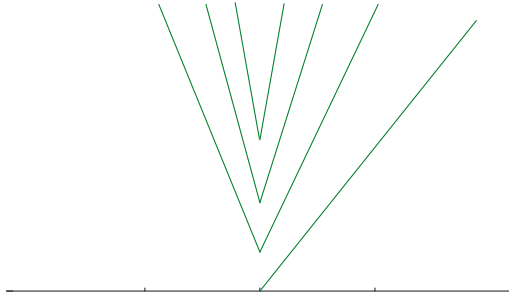
$$h_t \quad ($$



$h_{\pm} = \frac{1}{16}(r_{\pm} - s_{\pm})^2$  (10),  
 $s_{\pm}, r_{\pm} = \dots$  (14) (15).  
 $A = \dots$  (10)

**3. BBM approximation and the structure of the expansion shock**

$\dots$  (11)



where  $\sigma$  is the ratio of specific heats,  $\rho$  is the density,  $\mu$  is the dynamic viscosity,  $\kappa$  is the thermal conductivity,  $g$  is the gravity, and  $h$  is the enthalpy. The boundary conditions are

**4. Expansion shock for the Boussinesq equations**

In this section, we consider the expansion shock problem for the Boussinesq equations. The flow is assumed to be inviscid and irrotational, so the velocity potential  $\phi$  satisfies Laplace's equation in the region  $x > \delta$ . The boundary conditions at the shock  $x = \delta$  are given by (17) and (18). The velocity potential  $\phi$  is expanded in powers of  $\delta$ , and the resulting expansion is substituted into the Boussinesq equations. This yields a hierarchy of equations for the coefficients of the expansion. The leading-order equation is the Laplace equation for the potential  $\phi^{(0)}$ . The next-order equation is the Poisson equation for the velocity correction  $u^{(1)}$ . The resulting expansion of the velocity potential  $\phi$  is given by (13). The expansion of the velocity  $u$  is given by (14).

*4.1. Inner solution: first-order approximation*

In this subsection, we consider the inner solution of the expansion shock problem. The inner region is defined by  $x < \delta$ . The velocity potential  $\phi$  is expanded in powers of  $\delta$ , and the resulting expansion is substituted into the Boussinesq equations. This yields a hierarchy of equations for the coefficients of the expansion. The leading-order equation is the Laplace equation for the potential  $\phi^{(0)}$ . The next-order equation is the Poisson equation for the velocity correction  $u^{(1)}$ . The resulting expansion of the velocity potential  $\phi$  is given by (15). The expansion of the velocity  $u$  is given by (16).

$$r^{(1)} = \frac{1}{4\delta} (3r^{(0)} - s^{(0)}) r^{(0)} - \frac{1}{6\delta^2} (r^{(0)} - s^{(0)})$$

$$s^{(1)}$$



$$r^{(0)}, r^{(1)}, r^{(2)}, s^{(0)}, s^{(1)}, s^{(2)} \rightarrow 0, \delta \rightarrow 0, \rightarrow 0. \quad (15)$$

$$\dots \quad (21) \quad \dots \quad (20), \quad \dots$$

$$(\epsilon r^{(1)} \dots)$$

$$\frac{1}{4\delta} (3r^{(0)} \quad s^{(0)} \quad 3\epsilon r^{(1)} \quad \epsilon^2(3r^{(2)} \quad s^{(2)} \quad \dots)) (\epsilon r^{(1)} \quad \epsilon^2 r^{(2)} \quad \dots)$$

$$\frac{1}{6\delta^2} (\epsilon r^{(1)} \quad \epsilon^2 (r^{(2)} \quad s^{(2)} \quad \dots)) \quad (22)$$

$$(\epsilon^2 s^{(2)} \quad \dots) \quad \frac{1}{4\delta} (r^{(0)} \quad 3s^{(0)} \quad \dots) (\epsilon^2 s^{(2)} \quad \dots) \quad (23)$$

$$\frac{1}{6\delta^2} (\epsilon r^{(1)} \quad \dots)$$

B  $\ll k < \delta, \dots \quad (22)$

$$\left(\frac{\epsilon}{\delta}\right) : \frac{1}{\delta}$$

$$K \left( \frac{2a}{9a^2} \frac{ff'}{f''} \right) K. \quad (30)$$

$$a(\cdot) = \frac{A}{\frac{9}{2}AK - 1} \cdot f(\cdot) \quad B \left( \frac{B}{2K} \right). \quad (31)$$

$$B - 1 \cdot K = \frac{1}{2}. \quad (32)$$

$$a(\cdot) = \frac{A}{\frac{9}{4}A - 1} \cdot f(\cdot) \quad (33)$$

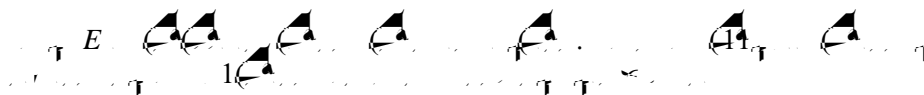
$$r^{(0)}(\cdot) = \frac{s^{(0)}}{3} \pm \frac{\sqrt{A}}{\frac{9}{4}A - 1} \quad (\leftarrow^2). \quad (34)$$

$$s^{(0)}(\cdot) = s^{(0)} \quad (\leftarrow^2). \quad (35)$$

$$r_{\pm} = \frac{s^{(0)}}{3} \pm \sqrt{A} \quad (\leftarrow^2). \quad (36)$$

$s_{\pm}$

$$\begin{aligned}
 > t \gg x = 0, \quad > \frac{1}{4}(s^{(0)} - 3r) \quad \text{BB} \quad (19). \\
 > \frac{1}{4} \quad \text{BB} \quad (18). \\
 > \text{BB} \quad 1 \quad > 5.
 \end{aligned}$$



$$r^{(c)}(\cdot) \sim 1 - \frac{\epsilon}{4} \left( \frac{1}{1 - \frac{9}{4}} \right) \\ \frac{\epsilon^2}{3} \left( C \frac{2 - 17 \cdot^2(\cdot) - D \cdot^2(\cdot) - E \cdot^2(\cdot)}{16(1 - \frac{9}{4})^2} \right).$$

$$s^{(c)}(\cdot) \sim 3 \frac{3}{4} \frac{\epsilon^2}{4} \left( C \frac{3 \cdot^2(\cdot)}{16(1 - \frac{9}{4})^2} \right).$$



$$\begin{aligned} r^{(1)} &= \frac{1}{4}(3r^{(0)} - s^{(0)})r_X^{(0)} = 0, \\ s^{(1)} &= \frac{1}{4}(r^{(0)} - 3s^{(0)})s_X^{(0)} = 0. \end{aligned} \tag{51}$$

$$\begin{aligned} r^{(2)}(X) &= 3 \left[ \frac{3}{4} r^{(1)} - \frac{1}{32} r^{(0)2} \right], \dots \\ r^{(3)}(X) &= 1 \left[ \frac{1}{4} r_1(X) - \frac{1}{96} r_2(X) \right], \dots \\ s^{(2)}(X) &= \dots \\ s^{(3)}(X) &= \dots \end{aligned} \tag{52}$$

From (58), (50), and (59), we have

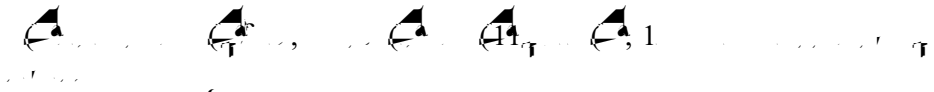
$$r_2(X) \underset{X \rightarrow 0^\pm}{\sim} F_2^\pm \frac{1}{\left(1 - \frac{9}{4}\right)^2} \underset{\pm}{\rightarrow} r^{(2)}(X) = \frac{1}{24\left(1 - \frac{9}{4}\right)^2}. \quad (60)$$

For  $F_2$ ,  $F_2 < 24$ , we have

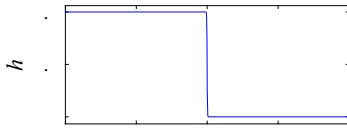
$$r_2(X) = \frac{1 - 3X}{24\left(1 - \frac{9}{4}\right)^2}. \quad (61)$$

From (61), we have

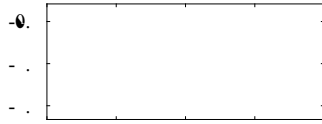
$$r^{(1)}(X) = 1 - \left( \frac{1}{4} - \frac{X - 3X}{1 - \frac{9}{4}} \right) \\ \frac{\omega^2}{24} \left( \frac{1}{4} - 1 - 3 \right)$$

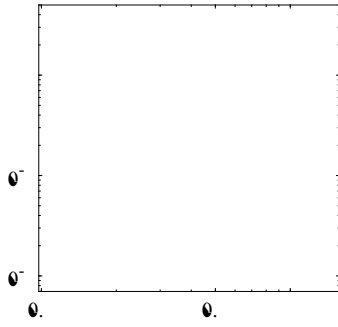


$$r^{(i)}(X, \cdot) = 1 \quad \left\{ \begin{array}{l} \\ \\ \\ \\ \end{array} \right.$$

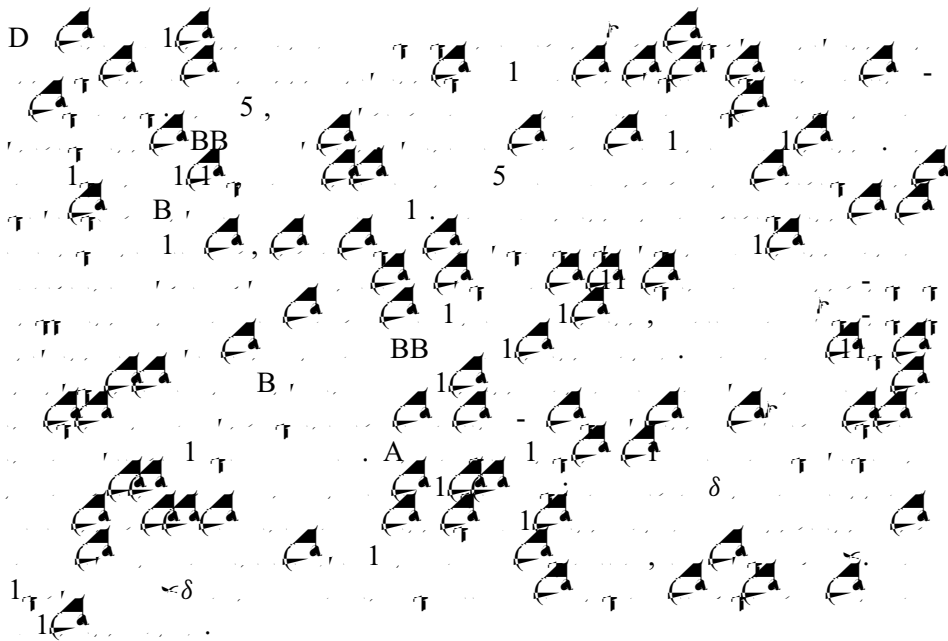








### 6. Discussion



Acknowledgments

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Appendix

A.1. Let  $u(x, t)$  and  $h(x, t)$  be the solutions of the initial-boundary value problem (1.1)–(1.4) with  $u(x, 0) = u_0(x)$  and  $h(x, 0) = h_0(x)$ . Then, the following lemma holds.

$$g_t - (ug)_x - (h^2)_x = 0, \quad (A1)$$

$$g_t - (u^2)_x - g_x = \frac{1}{3} u_{xxt} = 0.$$

B. Let  $h(\pm L, t) = h_{\pm}$  and  $u(\pm L, t) = u_{\pm}$  for  $t \geq 0$ . Then, the following lemma holds.

$$f(u)_{x_n} = ik_n f(u)_{g_n}, \quad n = N-2, \dots, N-2-1. \quad (A2)$$

Let  $h(x_n, t) = h$  and  $g(t) = \frac{1}{2L}(h - h^2)(x_n - L)$ .

$$h(x_n, t) = h + \int_0^t g(t - \tau) d\tau = \frac{1}{2L}(h - h^2)(x_n - L). \quad (A3)$$

Let  $g_n(t) = \begin{cases} \frac{2L}{N} \sum_{m=N-2}^{N-2-1} x_m g(x_m, t) & n = 0 \\ \frac{g_n(t)}{ik_n} & n = 0 \end{cases} \quad (A4)$

Let  $g(x, t) = \int_0^L x g(x, t) dx$ . Then, the following lemma holds.

The diagram consists of several rows of elements. The top row features a series of arrows pointing right, with the number '1' appearing above several of them. Below this, there are more arrows and symbols, including the expression  $0.002$  and  $xx < 3$ . A central part of the diagram contains the label '(A1)'. To the right, there is a large number '120' followed by a parenthesis '('. Below these, the expression  $N = 2^{14}$  is visible. The bottom row shows a sequence of arrows pointing right, with the number '1' appearing above one of them. The overall structure suggests a flow or a sequence of operations or states.