Discrete Radon Transform

GREGORY BEYLKIN

| 4 | AbstractThis paper describes the discrete Radon transform (DRT) | various discretizations of Radon's inversion formula. We |
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show that DRT can be used to compute various generali- where

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BEYLKIN: DISCRETE RADON TRANSFORM

This is the key observation which follows from the periodicity condition (i). (Discussion of properties of the block-circulant matrices can be found in [27] for exam-

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It follows from (4.6) that if $\sigma = 1$, matrices R_m are given by

$$(R_m)_{jl} = \delta_{m,jl}.$$

where $j = 0, \pm 1, \cdots, \pm J$. This transform reduces to the ordinary DFT for $\alpha = 1$ and L = J. We consider now the following problem: given α and $\hat{w}_{\alpha}(j)$ for j = 0, $\pm 1, \cdots, \pm J$, find w(l). To solve this problem, we ap-

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| | = $N/k_0(2J + 1)$, where $k_{\min} \le k_0 \le k_{\max}$), estimates of the eigenvalues of the matrix $\hat{H}_{m}(k)$ can be obtained using | Inversion formula (6.1) also implies the discrete Par- seval's identity. In the continuous case. Parseval's iden- |
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| | One can see now that the expression in (4.8) is a discrete explored of the kernel in the inner integral in (7.2) . If we | 0.000 | 0.025 | 0.050 | 0.075 | 0.100 | 0.125 | 0.150 |
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| | | | | | | | | 3 | Lemma 1 and Lemma 2 are essentially similar. Their |
| | 2.0 | | | | | | | 5.0 | proof is elementary. We use the notation of Lemma 1. |
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