Mathematical and Computational Methods in Seismic Exploration and Reservoir Modeling

(where $\sigma = \sigma_R + i\sigma_I$, $\sigma_R \neq 0$ is a complex scalar, x and k are n-dimensional real and complex vectors respectively). For each j, $\frac{\partial u}{\partial k_j}$ can be expressed [illegible]

To illustrate our approach, we consider a medium where wave propagation is described by the Helmholtz equation. Suppose the index of refraction in some region X is of the form $n^2(x) = n_0^2(x) + f(x)$, where $n_0(x)$ -- the background index of refraction--is known. Then the problem is to characterize the function $f(x)$ using observations of the (singly) scattered field on the boundary ∂X of the region X. The incident field is generated by a point source at the point η located outside the region of interest. Let the region X be three-dimensional, however, the specific dimension of X is not essential in our approach, and enters only as a parameter. We treat the case of a variable background index of refraction and arbitrary configuration of sources and receivers.

The linearized inverse scattering problem is formulated in terms of an integral equation of the first kind with an oscillatory kernel relating the singly scattered field to the perturbation $f(x)$

$$v_{sc}(k,\xi,\eta) = -k^2 \int_X G(k,\xi,x)f(x)G(k,\eta,x)\,dx \ , \qquad (1)$$

where ξ and η denote locations of receiver and source, respectively. The Green's function G is the solution of the equation

$$\phi(x,\xi,\eta) = \phi(\xi,x) + \phi(\eta,x) ,$$

and

$$a(x,\xi,\eta) = A(\xi,x)A(\eta,x) ,$$

then within the geometric optics approximation $v_{sc}(k,\xi,\eta) = -k^2(Rf)\hat{}(k,\xi,\eta)$, where $\hat{}$ denotes the one-dimensional Fourier transform of (3) with respect to variable t.

Now the problem of solving (1) can be cast as an inversion problem for GRT. The inversion of the GRT requires the introduction of Fourier Integral Operators (FIO). A special role is played by a FIO of the form $F = R^*KR$. Here, R denotes the GRT, R^* is an operator dual to R, and K is a one-simensional convolution operator. R^* is also known as the Generalized Backprojection Operator (GBO). By properly choosing the convolution operator K and the weight function of the GBO the problem of inverting the GRT is reduced to that of solving a Fredholm integral equation.

Exploiting the fact that F is "almost" the identity operator we rigorously establish a class of migration algorithms as approximate solutions of the linearized inverse scattering problem. We prove that

$$F = I + T_1 + T_2 + \dots ,$$

where $T_1, T_2, \dots$ belong to increasingly smooth classes of pseudo-differential operators, $T_j \in L^{-j}(X)$, for $j = 1,2,\dots$ and [illegible]

the number of derivatives of a function to which it is applied. The approximation amounts to using only the first term of an asymptotic expansion for the solution of the integral equation (1), which in terms of GRT can be written as

$$R^*KR \simeq I.$$

Due to the nature of the asymptotics we give a precise meaning to what is reconstructed by this first order inversion for arbitrary configurations of sources and receivers, including the case of limited [illegible]

Our method yields an algorithm for recovering these discontinuities