28 (2010) 131 149

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a r t i c l e i n f o

Article history:
15 2009
15 2009
2 2009

Keywords:
0
0
0

a b s t r a c t

B (2005) 16

> 0 … $\delta > 0$, … $r^{-\beta}$, $\beta > 0$, …

$$\left| r^{-\beta} - \sum_{m=1}^{M} w \right.$$

1. *Let us assume that* (4) *holds. For any* > 0 *and* $t_0 \in \mathbb{R}$, *we have*

$$\left| \int_{\mathbb{R}} f(t)\,dt - h \sum_{n\in\mathbb{Z}} f(t_0 + nh) \right| \leqslant \tag{6}$$

provided that the Fourier transform of f *satisfies*

$$\left|\hat{f}(\xi)\right| \leqslant c_1 e^{-q|\xi|}, \tag{7}$$

for some positive constants c_1, q *and step size* $h \leqslant q/$ $(2c_1 \;^{-1} + 1)$ *or, alternatively,*

$$\left|\hat{f}(\xi)\right| \leqslant \frac{c_2}{|\xi|^q}, \quad \text{for } |\xi| \geqslant R, \tag{8}$$

for some positive constants c_2, R, q *and step size* $h \leqslant$ $\{1/R,\; ^{1/q}(2c_2\zeta(q))^{-1/q}\}$, *where* $\zeta(q)$ *is the Riemann Zeta function.*

(5), h $\sum_{n\neq 0} |\hat{f}(\frac{n}{h})| \leqslant$ (7),

$$\sum_{n\neq 0} \left| \hat{f} \; \{ \right.$$

$$S_{\infty}(r) = \frac{h}{(\beta)} \sum_{n \in \mathbb{Z}} e^{\beta(t_0+nh)} e^{-e^{t_0+nh} r}. \tag{13}$$

$h = h(\ ,\beta)$

$$\sum_{n \neq 0} \frac{|\ (\beta + 2\pi i \frac{n}{h})|}{(\beta)} < \ . \tag{14}$$

3. *Given* $\beta > 0$ *and* $0 < \ \leqslant 1$*, for any step size* h *such that*

$$h \leqslant \frac{2\pi}{3 + \beta \ (\ 1)^{-1} + \ ^{-1}}, \tag{15}$$

and any $t_0 \in \mathbb{R}$ *we have*

$$\frac{|r^{-\beta} - S_{\infty}(r)|}{r^{-\beta}} \leqslant \ , \quad \text{for all } r > 0, \tag{16}$$

where S_{∞} *is given in* (13).

$S_{\infty}(r)$ (16) r. ,

β,3 9 8 ( ) 4 0 .

4. *For all* $r > 0$,

$$S_F(r) < S_\infty(r) < (\ +1)r^{-\beta}.$$

, $S_F(r)$, *on the whole positive axis*, 12 . 4 9 .

fi f (10) r β. (16). □

S_F β, δ ,

5. *For any* $\beta > 0$, $\delta > 0$, *and* 1 / 7 1 9.7304 0 0 9.7304 303 *ou8309o94 0 TD 0.2518 f4p -33.1058 -2.866 TD 0.0004 0 9.*

B M N $r \in [\delta, 1]$. $f_{\beta,r}(y)$ (9) (10) $y_0 =$ $(r^{-1}\beta)$. M $t_M \leqslant$ β $f_{\beta,r}(y)$ $(-\infty, t_M)$ $r \in (0, 1]$ $\mathcal{T}_M(r)$

$$\mathcal{T}_M(r) \leqslant \frac{r^\beta}{(\beta)} \int_{-\infty}^{t_M} e^{-re^y+\beta y}\,dy \leqslant \frac{1}{(\beta)} \int_{-\infty}^{t_M} e^{-e^y+\beta y}\,dy \tag{27}$$

$$= \frac{1}{(\beta)} \int_0^{e^{t_M}} e^{-s} s^{\beta-1}\,ds = 1 - \frac{(\beta, e^{t_M})}{(\beta)}, \tag{28}$$

$$(\beta, x) = \int_x^\infty e^{-s} s^{\beta-1}\,ds$$

t_M δ.
N

$$t_N \geqslant \ \left(\delta^{-1}\beta\right), \tag{29}$$

fi

$$\mathcal{T}^N(r) \leqslant \frac{r^\beta}{(\beta)} \int_{t_N}^\infty e^{-re^y+\beta y}\,dy = \frac{1}{(\beta)} \int_{re^{t_N}}^\infty e^{-s} s^{\beta-1}\,ds,$$

$r \in [\delta, 1]$

$$\mathcal{T}^N(r) \leqslant \frac{(\beta, \delta e^{t_N})}{(\beta)}. \tag{30}$$

$$\lim_{x\to 0} \frac{(\beta, x)}{(\beta)} = 1 \qquad \lim_{x\to\infty} \frac{(\beta, x)}{(\beta)} = 0,$$

fi t_* t^* ,

$$1 - \frac{(\beta, e^{t_*})}{(\beta)} = \ , \tag{31}$$

$$\frac{(\beta, \delta e^{t}\ )}{(\beta)} = \ . \tag{32}$$

M_* N^* fi S_F, t_*, t^*, h t_0, $(t_* - t_0)/h$ $(t^* - t_0)/h$

7. *For all $\beta > 0$, $\delta > 0$ and $1/e \geqslant \ > 0$, the solution t_* of (31) does not depend on δ and satisfies*

$$t_* \geqslant \ \frac{(1+\beta)}{\beta} = \frac{1}{\beta} \ + \ (1+\beta)^{\frac{1}{\beta}}. \tag{33}$$

The solution t^* of (32) has a weak dependence on and satisfies

$$t^* \leqslant \ \delta^{-1} + \ \ ^{-1} + \ \beta + \frac{1}{2}. \tag{34}$$

t_* t^*, (33) (34). S_F 5. 7 A A.2. (9) (10) $-\infty$ $+\infty$ S_F e^{hn} (21) (n). A 16 (21) 3 .

8. *For any $\delta > 0$, and > 0, there exist a step size h and a positive integer M such that*

$$\left|e^{-xy} - G_e(x, y)\right| \leqslant \quad , \quad \text{for } xy \geqslant \delta, \tag{41}$$

where

$$G_e(x, y) = \frac{hx}{2\sqrt{\pi}} \sum_{j=0}^{M} e^{- \ x^2}$$

be an approximation of the kernel by Gaussians valid for $\delta \leqslant$

A $u(z,\cdot)$ $z>0$, $u_{zz} + \ u = 0$ $u(0,\cdot) = u_0(\cdot)$,

$$u(z,\cdot) = \int_{\mathbb{R}^d} \mathcal{P}(z,\cdot - \ )u_0(\ )\,d\ , \quad z \geqslant 0, \tag{56}$$

$$\mathcal{P}(z,\cdot) = \frac{2}{\omega_d}\frac{z}{(z^2 + \|\cdot\|^2)^{(d+1)/2}}$$

0 3, > 0 $z > 0$, 0

$$S_\infty\left(z^2 + \|\cdot\|^2\right) = \frac{zh}{\pi^{(d+1)/2}}$$

$0 \leqslant n \leqslant 2N$. , $a > 0$ $[0,1]$ ffi w_m (66) $\eta_m = \frac{2N}{a} t_m$, $h(x)$ $x \in [0, a]$,

$$\left| h(x) - \sum_{m=1}^{M} w_m e^{-\eta_m x} \right| < \quad , \tag{67}$$

h (66) . (66) :

- B $(N+1) \times (N+1)$ 0 $_{kl} = h_{k+l}$ $h_n = h(a \frac{n}{2N})$, $0 \leqslant n \leqslant 2N$.
- $= (u_0, \ldots, u_N)$, $= \sigma$, σ 0 (16, . 22); σ fi $M+1$ $\sigma_0 \geqslant \sigma_1 \geqslant \cdots \geqslant \sigma_M$, σ_M $\sigma_M/\sigma_0 \approx$, , $M = \mathcal{O}(\quad ^{-1})$ $M \ll N$.
- C γ_m $u($

$x =$ $^{-1}$. , β

$$\tilde{t}^* = \quad \delta^{-1} + \quad ^{-1}. \qquad (70)$$

$\beta > 1$,

$$\frac{(\beta, x)}{(\beta)} \leqslant \frac{e}{(\beta)} x^{\beta-1} e^{-x} \leqslant \quad ,$$

fi (69)

$$x > d(\beta - 1) \qquad (71)$$

$d = e/(e-1) \approx 1.582$, , 14, x

$$x \geqslant d$$

A.3. Proof of Theorem 5

2.1.1 (20) h (15) t_* t^* 7. $t_0 = 0$

$$\tilde{h} = \frac{10}{2 \quad {}^{-1} + \quad \beta + 2}.$$

,

$$\frac{\tilde{t}^* - \tilde{t}_*}{\tilde{h}},$$

$\tilde{t}^*$, $\tilde{t}_*$ (33) (34).

$$\tilde{t}^* - \tilde{t}_* = \quad \delta^{-1} + \frac{1}{\beta} \quad {}^{-1} + \quad {}^{-1} + \quad \left(\frac{\beta + 1}{(1+\beta)^{\frac{1}{\beta}}} \right) + \frac{1}{2}$$

$$\leqslant \quad \delta^{-1} + \frac{1}{\beta} \quad {}^{-1} + \quad {}^{-1} + \frac{3}{2}$$

15.

(23) (27) (28).

15. *Let* $g(x) = \frac{(x+1)^{\frac{1}{x}}}{x+1}$ *for* $x > 0$

[33] [illegible] B [illegible] 0 [illegible] : A [illegible] 0 [illegible] C [illegible] 121 (7) (2004) 2866–2876.
[34] [illegible] B [illegible] 0 [illegible] C [illegible] 121 (14) (2004) 6680–6688.
[35] [illegible] 0 [illegible] A [illegible] C [illegible] 20 (2) (1999) 699–718 ([illegible]).