Fast algorithms for Helmholtz Green's functions

B G. EG . BE I, *, CH. HE. . C. A, D CA

2 , , 030 -0 2 ,

H G '

G ,

G ,

H G , ,

= K ,

B. E (1921)

(1.6) (G & (1980) (1998)

Proposition 2.1. ([illegible]) [illegible] $\in \mathcal{S}(\mathbb{R}\ )$, Λ [illegible]
[illegible], Λ^* [illegible]

3. Quasi-periodic Green's function via absolutely convergent series

[illegible] G [illegible] (1.6) [illegible]

( ), (1.1) (1.4) (1.5).

Proposition 3.1. *(3.1) (1.2) (1.3)* > 0, $\neq 2$ $\boldsymbol{d}$K , $\boldsymbol{d} \in \Lambda^*$ $\in \mathbb{R}$. ≥ 2.

F (3.1)

$$\text{F} \quad \text{C} = \frac{1}{} \sum_{\boldsymbol{d} \in \Lambda^*} \frac{\left(\frac{\text{K}\,2\ \boldsymbol{d}\text{K}\ {}^2\text{C}\ {}^2}{4\ {}^2}\right)}{2\ \boldsymbol{d}\,\text{K}\ {}^2\,\text{K}\ {}^2} \ \$\,2\ \boldsymbol{d}\text{K} \quad \text{K}\ \$ = \text{K}\ \$ \quad \text{F} ,$$

3 3 1

$^2 \ \$\boldsymbol{d} = 1$ $\in \Lambda$ $\boldsymbol{d}$

$$= \frac{1}{2}$$

Remark 3.3.

G

I (3.12), 2.1

> .

$$\frac{1}{2^{3/2}} \quad 0 \quad \frac{C^{2}}{4^{2}} \quad \in \Lambda \quad \$ \; K \; C^{2}\,{}^{2}$$

$$= \frac{1}{2} \quad 0 \quad \frac{C^{2}}{4^{2}} \quad d \in \Lambda^{*} \quad \$\,2\;dK \quad K \frac{2\;d\,K^{2}}{4^{2}} \quad \frac{}{3},$$

Λ^{*} . B

,

$$\frac{1}{2} \quad d \in \Lambda^{*} \quad \$\,2\;dK \quad 0 \quad \frac{K\,2\;d\,K^{2}\,C\;C^{2}}{4^{2}} \quad \frac{}{3}$$

$$= \frac{1}{} \quad d \in \Lambda^{*} \quad \frac{\$\,2\;dK \quad \overline{K\,2\;dK^{2}}}{}$$

ET 2 4 7 . 5 . 5 6 0 . 5 4 6 9 0 j/ F 2 1 1 7 . 6 7 1 1

4. Fast convolutions with Green's function

G (3.1) (3.2) (3.2) G G ( )

(3.4).

F F

$$\tilde{F} = \frac{1}{\ldots} \sum_{\substack{d\in\Lambda^* \\ 2\, d \leq}} \frac{\left(\frac{2\, d\,^2 C^2}{4^2}\right)}{2\, d\,^2 \,^2} \ldots 2\, d \tag{4.1}$$

> 0 > 0

F

(3.3)

G F > 0

(3.2)

$$\sum_{\substack{\in\Lambda \\ \leq}} \ldots C,$$

B. ( ) G ,

$$= \sum_{=1} \ldots^2, \tag{4.2}$$

> 0 > 0. (B. ( )). (4.2),

$$\tilde{\ } = \sum_{\substack{\in\Lambda \\ \leq}} \ldots C. \tag{4.3}$$

C (4.1) (4.3), G ’

$$= \quad C \; \tilde{}_{F} \; . \qquad (4.4)$$

: ( )

( )

(4.2).

$\tilde{}_{F}$ F

$$\tilde{}_{F} \; *, \; = \frac{1}{} \sum_{\substack{d \in \Lambda^* \\ 2 \; d K \; \leq}} \frac{\left(\frac{K \; 2 \; dK \; ^2 C \; ^2}{4 \; ^2}\right)}{}$$

d dK 2 2 d K

> 0 > 1

$$\frac{1}{} \sum_{\substack{\boldsymbol{d}\in\Lambda^* \\ 2\ \boldsymbol{d}\mathrm{K} >}} \frac{\left(\frac{\mathrm{K}\,2\ \boldsymbol{d}\mathrm{K}\ ^2\mathrm{C}\ ^2}{4\ ^2}\right)}{2\ \boldsymbol{d}\mathrm{K}\ ^2\mathrm{K}\ ^2} \leq \frac{}{3}$$

, ,

$$\left\| \ \mathrm{F} \quad \mathrm{K}\tilde{\ }\ \mathrm{F} \ \right\|_{1} \leq \frac{}{3}. \tag{4.8}$$

$\mathrm{K}\tilde{\ }$

$| \quad |^{1}$

(3.1) ≥

$$\frac{\left(\frac{K\ \ {}^2K\ {}^2}{4\ {}^2}\right)}{{}^2K\ {}^2} \leq \frac{}{{}^2\ {}^2K\,1}.$$

(4.16), (3.2)

A > 1, F ∼3,

Remark 4.2. D

E ' (C (1978) (1986) = 0).

(2006; 2006 B

). F

( )
F
B (2008), F
$\mathcal{O}($ $)$ ∼ . A ,
∼ C 2 K1 .

A :

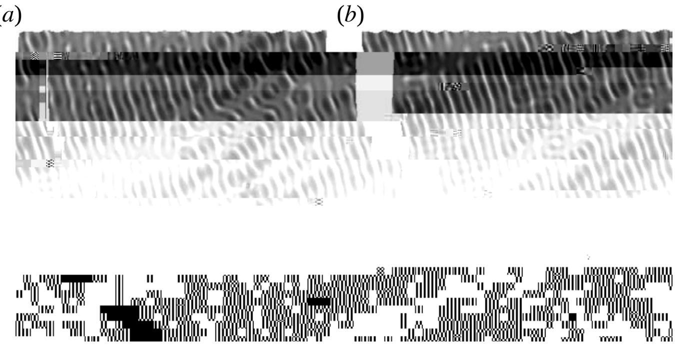
( ) B (2008),
(4.7) $\mathcal{O}$ $ $ $. A ,
G (G & 1991; 1991;
G & 1998) ,
. A $ $ ∼ K1 3,
(,
B (2008)
G). (4.7) =0
(4.7) $\mathcal{O}$ C 3 K1),
3
( ) F F
2 $\boldsymbol{d}$K ≤
K1
(3.1). G
(4.5), FF (D & 1993; B 1995;
& G 2005) $\mathcal{O}$ C C $\mathcal{O}$, $\mathcal{O}$ C
4 K1 , 4

F ,

(4.17) G ’
. B
G ’
. F
, G ’
(4.17) (4.17) ’
$= \Delta$ C (2000, (17)) Δ
(1998, (2.49), (2.53) (2.54))
I 1, (4.17)
(4.4) $\approx 10^{-9}$. $= 0$
($\approx 10^{-9}$) 4.1.

$$= \frac{2\alpha}{} \ldots {}^{3} \ldots \alpha \ldots C \ldots {}^{2} \quad \in \Lambda \ldots = 1 \tag{4.18}$$

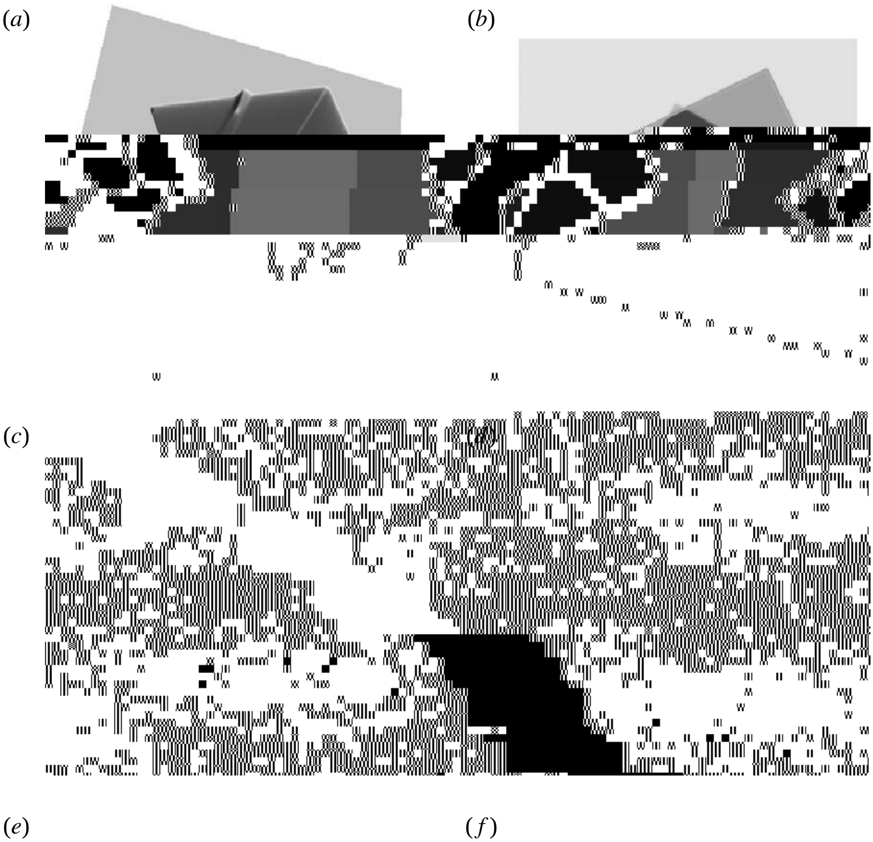
$\alpha = 300$, $= (1/3, 4/7)$, ${}_1 = (0, 0)$, ${}_2 = (1/10, 1/10)$ ${}_3 = (-3/$



F 3. A G = (3, 5) = 100 $_1$ = 1, 0 $_2$ = 1/2, 3/2 K1/2, 1/2 ! K1/2, 1/2 : ( ) ( ) .

F . I 2, G = 10^{K11}, 2 2 ≈ 1.76 2 ≈ 1.31$10^3. 4.1.

G ' . I 3, G : ( ) F ( ) G ' . I 4, G ' ( )



5. Green's functions with boundary conditions on simple domains

F G .
I , G
F
F , D
$= K1/2, 1/2 \; ! \; K1/2, 1/2$.
G , G (2
),

$$\Delta C 4^2 = K$$

(1.3) $= 0$. G

$$_1, _2 = K \frac{1}{4} \sum_{1=K\infty}^{\infty} \sum_{2=K\infty}^{\infty} {}_0 \; 2 \quad {}_1 C \; {}_1^2 C \; {}_2 C \; {}_2^2 ,$$

G , (1.6),
G , $_1 = 1, 0$ $_2 = 0, 1$.
(3.4),

$$_1, _2 = \frac{1}{2} \sum_{\in \mathbb{Z}^2}^{\infty} 2 \quad K \; C \; {}^2 \frac{2}{4} C \; 4^2 \; K2$$

$$C \sum_{\in \mathbb{Z}^2} \frac{K^2 \; {}^2 C^2}{}$$

$$= \left(K\frac{}{4} \quad K \quad C\,2 \quad {}^2\right) K \quad \left(K\frac{}{4} \quad C \quad C\,1\,C\,2 \quad {}^2\right). \qquad 5.3$$

(5.2)

$$\tilde{D} \quad , \quad , \quad = \quad \sum_{=1} \quad \int_{{}_1^2 C \; {}_2^2 \leq} \quad {}^{1/2}_{K1/2} \quad , \; {}_2 \quad 2, \; 2$$

$$! \quad {}^{1/2}_{K1/2} \quad , \; {}_1 \quad 1, \; 1 \quad 1, \; 2 \quad 1 \quad 2,$$

§4.

I F , , >1

(4.8)

$$\tilde{D}_F \quad 1, \; 2, \; 1, \; 2 \quad = \quad \int_{2 \quad {}_1^2 C \; {}_2^2 \leq} \quad \frac{\frac{K\;{}^2 \quad {}_1^2 C \; {}_2^2 \; C\;{}^2}{2}}{4\;{}^2 \quad {}_1^2 C \; {}_2^2 \; K\;{}^2} \quad {}_1 \; {}_1 C \quad {}_2 \; {}_2$$

$$! \left({}^{K \quad 1 \; 1} K \quad {}^{1 \; 1 C 1} \right) \left({}^{K \quad 2 \; 2} K \quad {}^{2 \; 2 C 1} \right). \qquad 5.4$$

$$\tilde{D}_F \quad , \quad , \quad = \quad \int_{2 \quad {}_1^2 C \; {}_2^2 \leq} \quad \frac{\frac{K\;{}^2 \quad {}_1^2 C \; {}_2^2 \; C\;{}^2}{2}}{4\;{}^2 \quad {}_1^2 C \; {}_2^2 \; K\;{}^2} \quad {}_1 \; {}_1 C \quad {}_2 \; {}_2$$

$$! \quad \hat{} \quad 1, \quad 2 \quad K \quad {}^1\hat{} \quad K \quad 1, \quad 2$$

$$K \quad {}^2\hat{} \quad 1, K \quad 2 \quad C \quad {}^{1 C \; 2}\hat{} \quad K \quad 1, K \quad 2 \quad , \qquad 5.5$$

(4.6). FF (5.5) §4 .

Remark 5.1. A

Remark 5.2. [illegible] G [illegible] D [illegible] $=3$ [illegible] . I [illegible]

A ( )

H 2003, 2004; 2004).

I

G

= 0.

H

A G

( )

( )

( ),

F E

F

G

F D -0612358, D E/
4000038129, D E DE-FG02-03E 25583 AF FA9550-07-1-0135.

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