

NAME: \_\_\_\_\_

SECTION: 001 at 9:05 am

**Instructions:**

1. Notes, your text and other books, cell phones, and other electronic devices are not



- (b) The number of calories in a cheeseburger on the lunch menu is approximately normally distributed with a mean of 434 and a variance of 49.
- (i) What is the probability that a randomly chosen cheeseburger will contain more than 420 calories?
  - (ii) Alex orders 8 cheeseburgers for a party. Assuming independence, find the probability that the total calories in the 8 cheeseburgers will exceed 3,450.
  - (iii) If the 8 cheeseburgers are served one at a time to 8 guests, what is the probability that the first guest to be served a cheeseburger with over 420 calories is the seventh guest? Explain.

- (c) Suppose the number of foreign fragments in a portion of peanut butter is a random variable with a mean of 3 and a variance of 3. Suppose a random sample of 50 portions of peanut butter are collected and the average number of foreign fragments in a portion is calculated. What is the probability of observing an average of 3.6 or more fragments?

**Problem 2.** (28 points) Let  $(X; Y)$  be jointly distributed random variables with conditional pdf given by:

$$f_{Y|X}(y|x) = \begin{cases} \frac{2y}{x^2} & 0 < y < x \\ 0 & \text{otherwise} \end{cases}$$

and marginal pdf of  $X$  given by:

$$f_X(x) = \begin{cases} 5x^4 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Find the joint pdf of  $X$  and  $Y$ .
- (b) Find the marginal pdf of  $Y$ :
- (c) Find  $E[Y|X]$  and use it to find the expectation of  $Y$ :
- (d) Find  $\text{Cov}(X; Y)$ .



**Problem 3.** (18 points) If  $X$  and  $Y$  have the following joint pdf, compute the joint density of  $U = \frac{X}{Y}$ ;  $V = Y$ :

$$f_{X;Y}(x;y) = \begin{cases} 2 & 0 < x < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

**Problem 4.** (26 points) Recall that if  $X \sim \text{Exp}(\lambda)$ ; then  $E[X] = 1/\lambda$  and  $\text{Var}(X) = 1/\lambda^2$ :

Recall that if  $Y \sim \text{Unif}(0; 1)$ ; then  $E[Y] = 1/2$  and  $\text{Var}(Y) = 1/12$ :

Mason has 200 batteries whose lifetimes are independent exponential random variables, each with a mean of 4 hours.

- (a) If the batteries are used one at a time, with a failed battery being replaced immediately by a new one, approximate the probability that there is still a working battery after 810 hours.
- (b) Suppose that the time it takes (in hours) to replace a failed battery is uniformly distributed over  $(0, .8)$ , independently. Approximate the probability that all batteries have failed before 1000 hours have passed.

**Bonus Problem.** (3 points) Let  $X$  be a random variable with mean 50 and variance 25. If  $X$  is the number of cars produced in a week at a particular auto manufacturing plant, find a lower bound on the probability that the production of cars in a week is between 30 and 70.

