

This exam is worth 150 points and has 6 problems.

Show all work and simplify your answers! Answers with no justification will receive no points unless otherwise noted.

Please begin each problem on a new page.

DO NOT leave the exam until you have satisfactorily scanned and uploaded your exam to Gradescope.

You are taking this exam in a proctored and honor code enforced environment. **NO** calculators, cell phones, or other electronic devices or the internet are permitted during the exam. You are allowed one 8.5" x 11" crib sheet with writing on two sides.

Remote students are allowed use of a computer during the exam only for a live video of their hands and face and to view the exam in the Zoom meeting.

0. At the top of the first page that you will be scanning and uploading to Gradescope, write the following statement and sign your name to it: "I will abide by the CU Boulder Honor Code on this exam." **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

1. [2350/072823 (20 pts)] A jellyfish swims along the path $\mathbf{r}(t) = t^2 \mathbf{i} + \frac{1}{2}t \mathbf{j} + e^t \mathbf{k}$ from $0 \leq t \leq 3$ catching plankton as it moves along. If the density of plankton in the water is given by $P(x; y; z) = y \sqrt{4x + z^2}$ g/m find the total amount of plankton the jellyfish caught. Include units in your final answer.

2. [2350/072823 (30 pts)] The force of the current in Penelope the platypus's river can be described by the vector field $\mathbf{F} = (2x + \tan y) \mathbf{i} + (x \sec^2 y) \mathbf{j}$. Penelope swims first along the curve C_1 to the base of a waterfall then later returns along the curve C_2 with

$$C_1: \text{the line segment from } (2; 1) \text{ to } (2; -1) \quad C_2: \text{the curve } x = 2 + \cos \frac{1}{2}y \text{ from } (2; -1) \text{ to } (2; 1)$$

(a) (10 pts) Directly calculate the work done by the current on Penelope along C_1 by evaluating an appropriate line integral.

(b) (10 pts) Find the potential function of \mathbf{F} .

(c) (5 pts) Using a theorem from Calculus 3, determine the work done by the current on Penelope along C_2 .

(d) (5 pts) Determine the total work done by the current along the union of the two paths: $C = C_1 \cup C_2$.

3. [2350/072823 (36 pts)] Consider the open surface cut from $z = 3 \sqrt{x^2 + y^2}$ where $3 \leq z \leq 0$.

(a) (5 pts) What quadric surface is this?

(b) (10 pts) Give a parameterization of the boundary curve, C , of this surface with a counterclockwise orientation when viewed from above.

(c) (5 pts) What portion of a plane shares the same boundary?

(d) (16 pts) Use Stokes' theorem to evaluate $\int_C 3yz \, dx + 7xy \, dy + z \, dz$.

4. [2350/072823 (18 pts)] Consider the closed boundary C made by the curves $x = y^2$ and $x = -y^2 + 2$ oriented counterclockwise. Compute the flux of \mathbf{H} through C if $\mathbf{H} = (e^{-y^2} + 3x^2; \ln x + 6; 1)$.

5. [2350/072823 (22 pts)] Consider a three dimensional solid, E , bounded within $S_1 : x^2 + y^2 + z^2 = 4$ and below $S_2 : z = 3 \sqrt{x^2 + y^2}$. Find the outward flux of $\mathbf{F} = (\sin y; x \ln(z + 1); z^2)$ through the boundary of E .

6. [2350/072823 (24 pts)] Write the word **TRUE** or **FALSE** as appropriate. No work need be shown. No partial credit given.

(a) The function $f(x; y) = 1 - x^2 - y^2$ is guaranteed to have a minimum value for all $x; y$ in the first octant.

(b) The function $g(x; y) = e^{xy}$ has a saddle point at the origin.

(c) The cross product of \mathbf{k} and the acceleration vector of the path $\mathbf{r}(t) = 2t \mathbf{i} + 5t \mathbf{j} + t^2 \mathbf{k}$ is never the zero vector.

(d) The line whose vector equation is $\mathbf{r}(t) = (1; 6; 2) + t(1; -3; 2)$ intersects the xz -plane at the point $(-1; 0; 6)$.

(e) $\lim_{(x;y) \rightarrow (1;0)} \frac{xy + x}{(x+1)^2 + y^2} = 2$ does not exist.

(f) The instantaneous rate of change of the function $g(x; y) = 4x^2 + 2x - 3y^2$ at the origin is largest in the $+\mathbf{j}$ direction.