

1. Let R be the region bounded by $y = 2 - x$ and $y = x^2$

(a) (5 points) Sketch the region. Be sure to label all axes, curves, and intersection points.

SOLUTION:

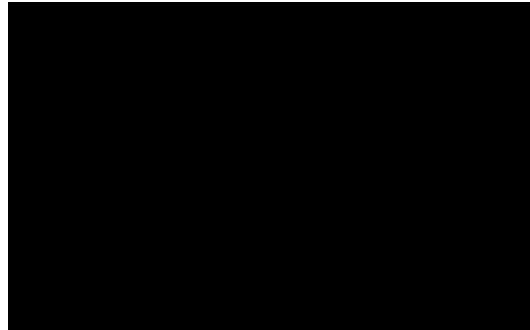


Figure 1: Bounded Area Region

(b) (7 points) Set up the dx integral(s) (integral(s) with respect to x) which, if evaluated, would give the area of the region. **DO NOT EVALUATE**.

SOLUTION:

$$\text{Area} = \int_{-2}^1 (2 - x - x^2) dx$$

(c) (7 points) Set up the dy integral(s) (integral(s) with respect to y) which, if evaluated, would give the area of the region. **DO NOT EVALUATE**.

SOLUTION:

$$A = \int_0^1 2 \sqrt{y} dy + \int_1^4 (2 - y) - \sqrt{y} dy$$

(d) (5 points) Find the area of the region from either (b) or (c).

SOLUTION:

Evaluation of either (b) or (c) will yield a result of $\frac{9}{2}$.

2. Consider the curve $y = \sin(x)$.

(a) (8 points) Use the Midpoint Rule with 4 sub-intervals to estimate the integral of this curve from $x = 0$ to $x = 4$.

SOLUTION:

Here, we know that $n = 4$; $a = 0$; and $b = 4$. Thus, we can say $\Delta x = \frac{(b-a)}{n} = 1$. We now divide the interval $[0,4]$ into

3. (a) (10 points) Evaluate the following integral $\int_0^3 \frac{1}{(x-1)^{2/3}} dx$ and determine whether it converges or diverges.

SOLUTION:

4. Evaluate the following integrals. Show all work!

(a) (12 points) $\int \frac{\ln(x)\ln(\ln(x))}{x} dx$

SOLUTION:

$$u = \ln(x)$$

$$du = \frac{1}{x} dx$$

$$\Rightarrow \int u \cdot \ln(u) du \Rightarrow u = \ln(u), du = \frac{1}{u} du; dv = u; v = \frac{u^2}{2}$$

$$\Rightarrow \int \frac{\ln(u)u^2}{2} - \int \frac{u}{2} du = \frac{u^2 \cdot \ln(u)}{2} - \frac{u^2}{4} + c$$

$$= \frac{(\ln(x))^2 \cdot \ln(\ln(x))}{2} - \frac{(\ln(x))^2}{4} + c$$

(b) (12 points) $\int x^3 \sqrt{4+x^2} dx$

SOLUTION:

$$x = 2 \tan(\theta); dx = 2 \sec^2 \theta d\theta$$

$$\Rightarrow \int \frac{8 \tan^3 \theta}{4 + 4 \tan^2 \theta} 2 \sec^2 \theta d\theta$$

$$\Rightarrow \int 32 \tan^3 \theta \sec^3 \theta d\theta$$

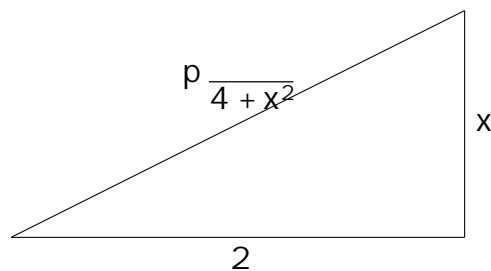
$$= 32 (\sec^2 \theta - 1) \sec^2 \theta \sec(\theta) \tan(\theta) d\theta$$

$$\Rightarrow \int u = \sec; du = \sec \tan \theta$$

$$= 32 \int (u^4 - u^2) du$$

$$= 32 \left[\frac{u^5}{5} - \frac{u^3}{3} \right] + c$$

$$= 32 \left[\frac{\sec^5(\theta)}{5} - \frac{\sec^3(\theta)}{3} \right] + c$$



$$= 32 \left[\frac{(\sqrt{4+x^2})^5}{5} - \frac{(\sqrt{4+x^2})^3}{3} \right] + c$$

(c) (12 points) $\int 3x^2 \, dx$