

1. (34 points) Find the requested information. The problems are unrelated.

(a) Evaluate $\int \frac{\tan^{-1}(x)}{x^2} dx$ (Hint: Start with IBP)

(b) Find y as a function of x given that $\frac{dy}{dx} = 2x^p \sqrt{1-y^2}$ and $y(0) = 1$

(c) Find the sum of the series $\frac{x^6}{3!} + \frac{x^{10}}{5!} - \frac{x^{14}}{7!} + \dots$

(d) For what values of x does the series $\sum_{n=0}^{\infty} \sin^2(x) \cos^{2n}(x)$ converge? Find the sum for those values of x

2. (16 points) Decide whether the following quantities are convergent or divergent. Explain your reasoning and any test you use.

(a) The sequence given by $a_n = 1 - \frac{\ln(3)^n}{n}$, for $n = 1; 2; \dots$

(b) $\int_1^{\infty} \frac{1}{x^2} \sqrt{1 + \frac{3}{x^3}} dx$

3. (12 points) Consider the series $\sum_{k=1}^{\infty} a_k$. Suppose the n th partial sum of the series is $2 - \frac{2}{n+1}$.

(a) What is a_3 ?

(b) Find a simple formula for a_n

(c) What does $\sum_{k=1}^{\infty} a_k$ converge to?

(d) What is the sum of the series $\sum_{k=1}^{\infty} k a_k$?

4. (25 points) Recall $\cosh(x) = \frac{e^x + e^{-x}}{2}$.

(a) Find the MacLaurin series of the hyperbolic cosine function.

(b) Find the interval of convergence for the power series from part (a).

(c) Find $T_3(x)$, the Taylor polynomial of order 3, of the hyperbolic cosine centered at $x=1$. Use the Taylor Remainder formula to find an upper bound for the absolute error if $T_3(x)$ is used to approximate $\cosh(1)$.

(d) Use the MacLaurin series (no L'Hopital!) to evaluate the following limit:

$$\lim_{x \rightarrow 0} \frac{\cosh(x) - 1 - \frac{x^2}{2}}{x^4}$$

5. (18 points) Suppose $f(x)$ equals the power series $\sum_{n=2}^{\infty} \frac{(n+1)(x+b)^n}{c^{2n}}$, where b and c are constants, and the series has an interval of convergence of $|x-b| < 2$.

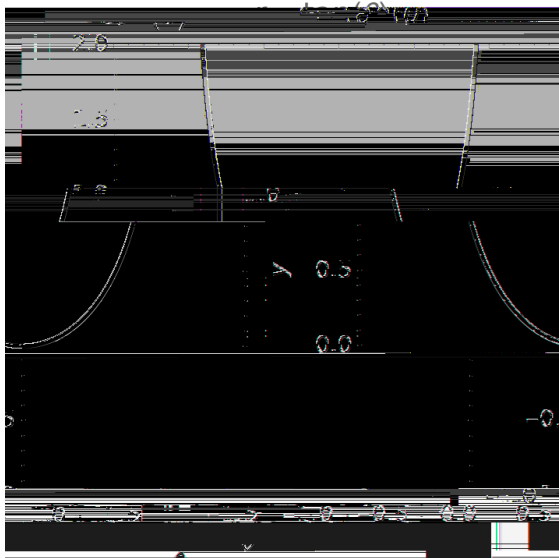
(a) Find the center and radius of convergence of the series.

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(b) Evaluate $\int g(x) dx$ as a power series.

(c) Given the interval of convergence, find possible values for θ . Justify your answer using appropriate test(s).

6. (25 points) For this problem, let $f(x) = \tan^{-1}(x)$ for $-2 < x < 2$. The polar graph (in the xy -plane) is given below. Answer the following questions.



(a) Find an equation for the tangent line at $x=4$.