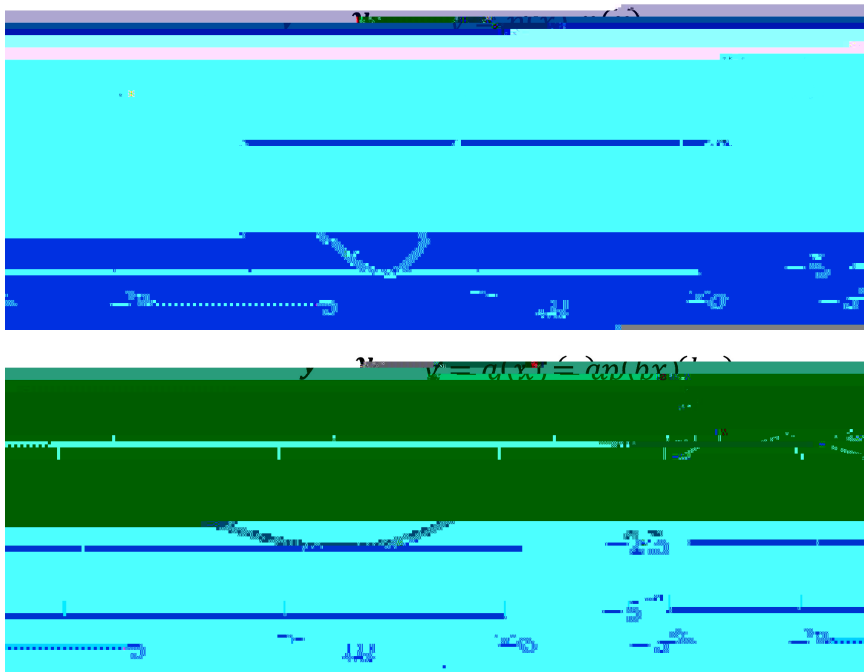


1. (20 pts) Parts (a) and (b) are not related.

(a) For $f(x) = \frac{1}{x-1}$ and $g(x) = \frac{1}{2-x}$

- (b) The graphs below depict the functions $y = p(x)$ and $y = q(x)$, where q is a transformation of p of the form $q(x) = ap(bx)$. Find the values of a and b .



Solution:

The vertical difference between the maximum and minimum values of the curve for $p(x)$ is $3 - (-5) = 8$, while the vertical difference between the maximum and minimum values of the curve for $q(x)$ is $1.5 - (-2.5) = 4$. Therefore, the curve for $q(x)$ has been constructed by applying a vertical contraction of a factor of 2 to the curve for $p(x)$. This implies that $a = 1/2$

The horizontal difference between the endpoints of the curve for $p(x)$ is $5 - 3 = 2$, while the horizontal difference between the endpoints of the curve for $q(x)$ is $10 - 6 = 4$. Therefore, the curve for $q(x)$ has been constructed by applying a horizontal expansion of a factor of 2 to the curve for $p(x)$. This implies that $b = 1/2$

Note that $q(x) = 0.5p(0.5x)$.

$$(b) \lim_{x \rightarrow 2} \frac{\sqrt{x+1} - \sqrt{3}}{x^2 + x - 6}$$

Solution:

Begin by multiplying the numerator and the denominator by the conjugate of the original numerator expression.

$$\lim_{x \rightarrow 2} \frac{\sqrt{x+1} - \sqrt{3}}{x^2 + x - 6} = \lim_{x \rightarrow 2} \frac{\sqrt{x+1} - \sqrt{3}}{x^2 + x - 6} \cdot \frac{\sqrt{x+1} + \sqrt{3}}{\sqrt{x+1} + \sqrt{3}}$$

x

$$= \lim_{x \rightarrow 2}$$

3. (30 pts) Consider the rational function $r(x) = \frac{3x^2 + 21}{x^2 - 4}$

- (c) Find the equation of each vertical asymptote of $y = r(x)$, if any exist. Support your answer in terms of your work in part (b).

Solution:

The finite value of $\lim_{x \rightarrow 5} r(x) = \frac{9}{8}$ determined in part (b) indicates that there is no vertical asymptote at $x = 5$.

The infinite limits $\lim_{x \rightarrow 3} r(x) = 1$ and $\lim_{x \rightarrow 3^+} r(x) = 1$ were determined in part (b). Either one of those limits being infinite is sufficient to conclude that the line $x = 3$ is a vertical asymptote of $y = r(x)$.

4. (20 pts) Parts (a) and (b) are not related.

(a) For what value of a is the following function $u(x)$ continuous at $x = 4$? Support your answer using the definition of continuity, which includes evaluating the appropriate limits.

$$u(x) = \begin{cases} \frac{x-4}{x^2-16} & ; x < 4 \\ \frac{1}{a-x} & ; x \geq 4 \end{cases}$$

Solution:

By the definition of continuity, $u(x)$ is continuous at $x = 4$ if $\lim_{x \rightarrow 4^-} u(x) = \lim_{x \rightarrow 4^+} u(x) = u(4)$.

$$\lim_{x \rightarrow 4^-} u(x) = \lim_{x \rightarrow 4^-} \frac{x-4}{x^2-16} = \lim_{x \rightarrow 4^-} \frac{x-4}{(x-4)(x+4)} = \lim_{x \rightarrow 4^-} \frac{1}{x+4} = \frac{1}{4+4} = \frac{1}{8}$$

$$\lim_{x \rightarrow 4^+} u(x) = \lim_{x \rightarrow 4^+} \frac{1}{a-x} = \frac{1}{a-4}$$

$$u(4) = \frac{1}{a-4}$$

Therefore, $u(x)$ is continuous at $x = 4$ if $\frac{1}{8} = \frac{1}{a-4}$