

1. (28 pts) The following problems are not related.

(a) Express the following as a polynomial: $(\sqrt{x-3} + 2)(\sqrt{x-3} - 2)$

Solution: Using the FOIL method produces a difference of squares.

$$(\sqrt{x-3} + 2)(\sqrt{x-3} - 2) = ($$

2. (22 pts) For the following, let point A be (2;1) and let point B be (-6;7):

(a) Find the distance between points A and B.

Solution: Apply the distance formula.

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-6 - 2)^2 + (7 - 1)^2} = \sqrt{(-8)^2 + 6^2}$$

$$D = \sqrt{100} = \boxed{10}$$

(b) Find a point-slope equation of the line passing through points A and B.

Solution:

The point-slope form of the equation of the line passing through a point $(x_1; y_1)$ is $y - y_1 = m(x - x_1)$. Since the line of interest passes through points A and B, the coordinates of either point can be used as $(x_1; y_1)$.

The parameter m , which represents the slope of the line, is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$

It does not matter which point is selected to be $(x_1; y_1)$ and which is selected to be $(x_2; y_2)$. The two possibilities produce the same slope value:

$$m = \frac{7 - 1}{-6 - 2} = \frac{1 - 7}{2 - (-6)} = \frac{3}{4}$$

Similarly, it does not matter which point is selected to be $(x_1; y_1)$ in the equation of the line:

Using point A: $x_1 = 2, y_1 = 1$: $y - 1 = \frac{3}{4}(x - 2)$

Using point B: $x_1 = -6, y_1 = 7$: $y - 7 = \frac{3}{4}(x - (-6))$) $y - 7 = \frac{3}{4}(x + 6)$

Both of the preceding results are valid point-slope representations of the line passing through points A and B.

(c) Find the midpoint of the line segment connecting points A and B.

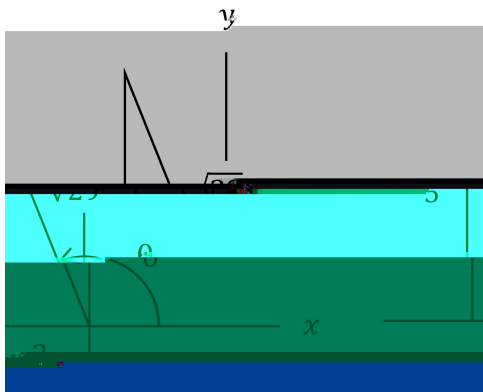
Solution:

3. (22 pts) The following problems are not related.

(a) If θ is on the interval $(\frac{\pi}{2}, \pi)$ and $\tan \theta = \frac{5}{2}$, find the value of $\csc \theta$.

Solution:

$\tan \theta = \frac{5}{2}$ and $\frac{\pi}{2} < \theta < \pi$ together imply the following orientation of θ :



The hypotenuse in the preceding figure was determined from the Pythagorean Theorem:

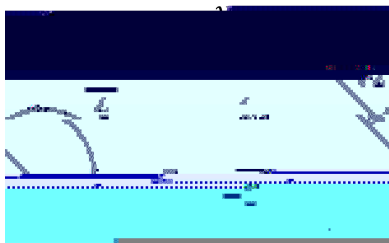
$$\rho \sqrt{(-2)^2 + 5^2} = \rho \sqrt{29}.$$

It follows from the figure that $\csc \theta = \frac{\rho \sqrt{29}}{5}$.

(b) Evaluate $\cos \frac{3}{4}$.

Solution:

Since $\frac{\pi}{2} < \frac{3}{4} < \pi$, the angle $\frac{3}{4}$ lies in Quadrant II, as drawn in the following figure.



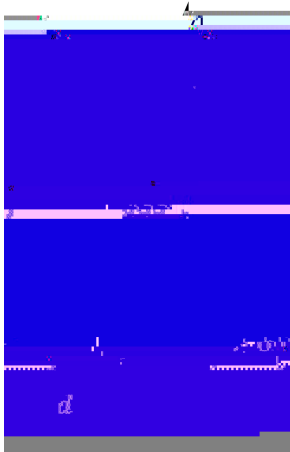
The reference angle is $\frac{3}{4} - \frac{\pi}{2} = \frac{1}{4}$, which is a 45° angle in the special 45°-45°-90° right triangle. The dimensions of such a triangle are proportional to 1, 1, and $\sqrt{2}$, which leads to the set of dimensions displayed in the figure.

It follows from the figure that $\cos \frac{3}{4} = \frac{1}{\sqrt{2}}$.

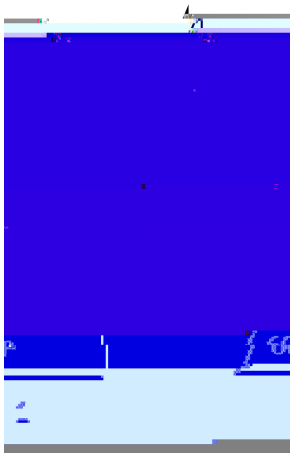
- (c) The height of a building is known to be 555 feet. A person standing a certain distance away measures the angle from their feet to the top of the building to be 60° . How far away is the person from the building? Express your answer in exact form and include the proper unit of measurement.

Solution:

The situation is depicted in the following figure:



The triangle in the preceding figure is the special $30^\circ - 60^\circ - 90^\circ$ right triangle, which is depicted below:



Since the two triangles depicted above are similar triangles, the following proportion of side lengths applies, which is also equal to $\tan 60^\circ$:

$$\tan 60^\circ = \frac{555}{d} = \frac{\sqrt{3}}{1}$$

Solving for d and applying the appropriate measurement unit of feet results in $d = \frac{555}{\sqrt{3}}$ ft

4. (28 pts) The following problems are not related.

(a) Find all values of x in the interval $[0; 2\pi]$ that satisfy the equation $\sin(2x) = \cos x$.

Solution:

The double-angle formula $\sin 2x = 2 \sin x \cos x$ can be substituted for the left-hand expression in the given equation:

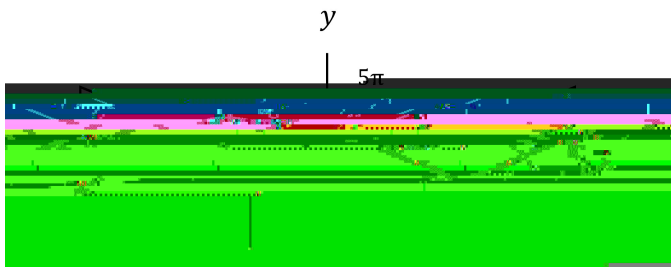
$$2 \sin x \cos x = \cos x \quad \Rightarrow \quad 2 \sin x \cos x - \cos x = 0 \quad \Rightarrow \quad \cos x(2 \sin x - 1) = 0$$

The Zero Factor Theorem indicates that $\cos x = 0$ or $2 \sin x - 1 = 0$.

$$\cos x = 0 \quad \Rightarrow \quad x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$2 \sin x - 1 = 0 \quad \Rightarrow \quad \sin x = \frac{1}{2}$$

The following figure can assist in evaluating $\sin x = \frac{1}{2}$:



Both triangles in the preceding figure are associated with an angle whose sine is $\frac{1}{2}$. The leg of length 1 and the hypotenuse of length 2 together imply that both triangles are special 30°-60°-90° right triangles. In such a triangle, the angle opp

(b) What is the radius of a circular sector having a central angle of 40° and an area of 4 ?

Solution:

The formula for the area of a circular sector is $A = \frac{\theta}{360} \pi r^2$

Solution: