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# An approximate renormalization for the break-up of invariant tori with three frequencies

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## Abstract

Renormalization theory provides a description of the destruction of invariant tori for Hamiltonian systems of  $1\frac{1}{2}$  or 2 degrees



dent resonances for  $\omega$ , then  $\omega = p$  where  $p$  is integral (remember the length of  $\omega$  is unimportant). A frequency  $\omega$  is *Diophantine* if there is a  $K \neq 0$  and  $\tau > 2$

tersection of each pair of resonances defines rational frequencies  $p_1 = [1, 0, 0]$ ,  $p_2 = [0, 1, 0]$ ,  $p_3 = [0, 0, 1]$ . The frequencies  $p_i$  also delineate the cone: it is the

such that  $\forall m \in \mathbb{Z}^3 \setminus 0, |m \cdot \omega| / |\omega| > K / |m|^\tau$ .

When  $A=B=C=0$ , the momenta  $(u, v)$  are constant in time and every orbit lies on a three torus. If  $\omega(u, v)$  is incommensurate, the orbit densely covers the torus. If  $\omega$  is Diophantine, then the KAM theorem implies that there is a torus with this frequency for small values of the amplitudes. We are interested

convex hull of the three vectors. We denote the cone by either of the matrices

$$M = \begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix}, \quad P = (p_1, p_2, p_3).$$

We assume  $\omega$  is inside the cone, i.e.  $\omega_i \geq 0$ .









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[19] R.S. MacKay and J. Stark, *Phys Lett A* 138 (1989) 113

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