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Abstract

Abstract text containing mathematical symbols and the year 2001.

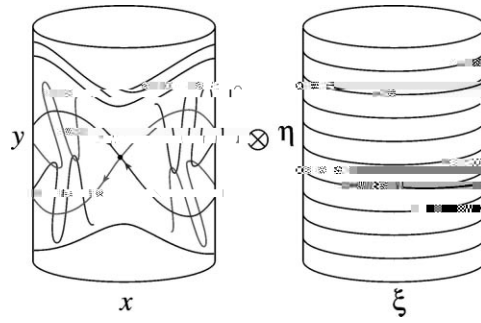
MSC: 37 40; 37 40; 37 50

Keywords:

1. Introduction

Introduction text containing mathematical symbols, equations, and the year 1964.

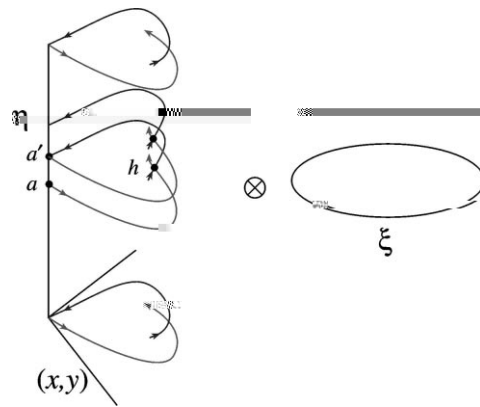
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(1) $w = h = 0$... (x, y)

$T^2 \times \mathbb{R}^2$... \mathbf{A}

$$x' = x + y', \quad y' = y - k(1 + h \dots) x, \quad \dots = -kh(\dots)$$



2. $\forall \epsilon > 0, \exists \delta > 0$, (1) $\forall w, h \neq 0, \forall \tau > 0, \exists C_a, C_{a'}, C_h$ such that $C_a, C_{a'}, C_h > 0$ and $h < C_h$.

(2) $\forall \epsilon > 0, \exists \delta > 0$, (1) $\forall w, h \neq 0, \forall \tau > 0, \exists C_a, C_{a'}, C_h$ such that $C_a, C_{a'}, C_h > 0$ and $h < C_h$.

$$\begin{aligned}
 & \text{The map } \mathcal{M} : (x, y) \mapsto (x', y'), \text{ is defined by} \\
 & x' = x + \frac{1}{2}y, \quad y' = y + V(x). \tag{4}
 \end{aligned}$$

For $n=1$, $V(x) = k_1(x)$, where $k_1(x) = \dots$

3. A t-t ab t

As $n=3, 4, 15$, $V(x) = k_n(x)$, where $k_n(x) = \dots$

$= *=-1,91.4887 -291.4887 -220.1(87 -)-17,1.4887 -2 1.4887 - \dots$

4. Conclusions

The results of this study show that the dynamics of the system are highly sensitive to the initial conditions and the parameters of the system. The system exhibits a rich variety of behaviors, including periodic, quasi-periodic, and chaotic dynamics. The bifurcation diagrams and phase portraits provide a clear visualization of these behaviors and their dependence on the parameters.

that C is not identically zero. Then given any $a < b$, there is a nonzero measure of initial states (x_0, x'_0) and a sequence $c_t \in (V)_+ \cup (V)_-$ such that the solution of (14) has momenta, $x_t = T_2(x_{t-1}, x'_t)$ satisfying $a < x_t < b$ and $T > b$ for some time T .

P ... $c_- \in (V)_-, c_+ \in (V)_+, \dots, x_t = c_{\pm} \dots (C(t)) = \pm 1.$
 (14) ...

$$\tilde{L}(x, x') = T(x, x') + W(x) + \tilde{C}(x),$$

... $\tilde{C} = V(c_{\pm}(x)) \quad C(x) \geq 0.$

... $\tilde{C}(x+2) - \tilde{C}(x) > 0, \dots = \tilde{L}(x+2, x'+2) - \tilde{L}(x, x'), \dots \mathbf{A} \dots \square$

4.2. Standard example

$$L(x, x', t, t') = \frac{1}{2} (x' - x)^2 + \frac{1}{2} (t' - t)^2 + k \dots x(1 + h \dots), \tag{15}$$

... $k > 0, h > 0, \dots \mathbf{A} \dots (1), \dots (15) \dots$

$t \rightarrow \dots$
 $^* = 2 - m$
 $(0, 2 - m) \dots (, 2 - m)$
 (16)
 $^* = (2m + 1)$
 $(0, ^*) \mapsto (, ^*)$

\mathbf{A}
 (16)

g
 g^*
 g
 (16)

$T^2 = \{(,) : 0 < , < 2\}$
 $^* = (t_{+1} - t)^* = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{t=0}^{T-1} (U'(t))$

\mathbf{A}
 $\langle ^* \rangle = \frac{1}{4 - 2} \int_{T^2} U'() = \frac{1}{2}$

(16)

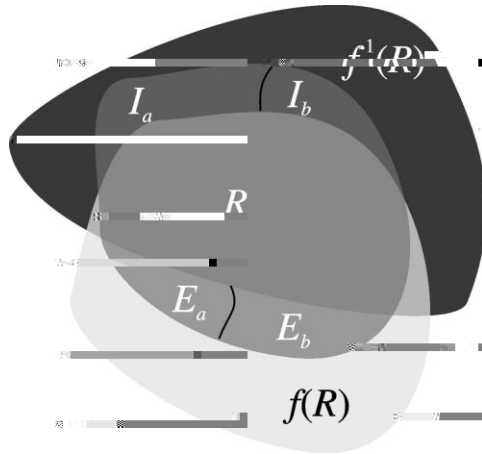


Fig. 4. The image of a set R .

where

$$E = \{z \in R : f(z) \notin R\} = R \setminus f^{-1}(R).$$

Since f is a contraction, $\mu(f^{-1}(R)) < \mu(R)$. Thus E is a non-empty set with positive measure.

$$\mu(E) = \mu(R \setminus f^{-1}(R)) = \mu(R) - \mu(R \cap f^{-1}(R)) = \mu(R) - \mu(f(R) \cap R) = \mu(R \setminus f(R)) = \mu(I). \quad (17)$$

Let S^t be the set of points in I that stay in R for t iterations of f . Then

$$S^0 = I, \quad S^t = f(S^{t-1}) \cap R = f(S^{t-1} \setminus E).$$

Since $S^t \subset R$, $\mu(S^t) < \mu(R)$. Also, $\mu(S^t) \rightarrow 0$ as $t \rightarrow \infty$. Thus $\mu(E) > 0$. The set E is a non-empty set with positive measure.

$$\mu(p(I_a) \cap E_b) = \mu(p(I_a)) - \mu(p(I_a) \cap E_a) \geq \mu(I_a) - \mu(E_a). \quad \square$$

A f_t R

$$I_t = R \setminus f_t(R), \quad E_t = R \setminus f_t^{-1}(R).$$

(17) f_t R k t S_k^t

$$S_k^k = I_{k-1}, \quad S_k^{t+1} = f_t(S_k^t \setminus E_t).$$

A S_k^t t $k \leq t$ R

$$\mu(S_k^t) < \mu(R) \quad (18)$$

L **a 4.** Let f_t be a sequence of measure-preserving homeomorphisms, and R a measurable set with incoming sets I_t and exit sets E_t .

5.2. Maps of the cylinder

$$\begin{aligned} & \int_C \langle \nabla \psi, \mathbf{v} \rangle = \int_C \langle \nabla \psi, \mathbf{v} \rangle \circ f^{-1} \\ & = \int_{f(C)} \langle \nabla \psi, \mathbf{v} \rangle \circ f^{-1} \\ & = \int_C \langle \nabla \psi', \mathbf{v}' - \mathbf{y}, \mathbf{x} \rangle \end{aligned}$$

Let A be an annulus in C bounded by the circles $\{y = a\}$ and $\{y = b\}$ where $a < b$. Let T and B be subsets of C such that $U \subset T$ and $D \subset B$.

$$U = \{z \in T : f^{-1}(z) \in B\}.$$

Let $D \subset B$ and T be subsets of C such that $U \subset T$ and $D \subset B$.

$$D = \{z \in B : f^{-1}(z) \in T\}.$$

$$\mu(U) - \mu(D) = \int_C \langle \nabla \psi, \mathbf{v} \rangle \circ f^{-1} - \int_C \langle \nabla \psi, \mathbf{v} \rangle \circ f^{-1}$$

Proposition 5. Suppose that f_t is a sequence of area and end-preserving homeomorphisms of the cylinder, and that the net flux $\int_C \langle \nabla \psi, \mathbf{v} \rangle \circ f_t^{-1} \geq \epsilon > 0$. Let A denote the annulus bounded by the circles $\{y = a\}$ and $\{y = b\}$ where $a < b$. Then, there is a set of positive measure of orbits that cross the annulus.

Proof. Let $U_t(a)$ and $D_t(a)$ be the sets $U_t(a) = \{z \in A : f_t^{-1}(z) \in D_t(a)\}$ and $D_t(a) = \{z \in A : f_t^{-1}(z) \in U_t(a)\}$. Let $U_t(b)$ and $D_t(b)$ be the sets $U_t(b) = \{z \in A : f_t^{-1}(z) \in D_t(b)\}$ and $D_t(b) = \{z \in A : f_t^{-1}(z) \in U_t(b)\}$. Let $E_t = U_t(a) \cup D_t(b)$ and $F_t = D_t(a) \cup U_t(b)$. Then $\mu(E_t) - \mu(F_t) = \int_C \langle \nabla \psi, \mathbf{v} \rangle \circ f_t^{-1} \geq \epsilon > 0$. \square

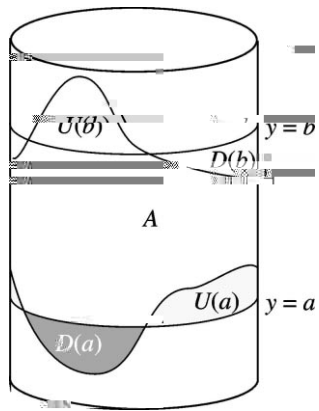


Fig. 5. A diagram illustrating the cylinder and the annulus A bounded by the circles $\{y = a\}$ and $\{y = b\}$. The regions $U(a)$, $D(a)$, $U(b)$, and $D(b)$ are shown, along with the annulus A.

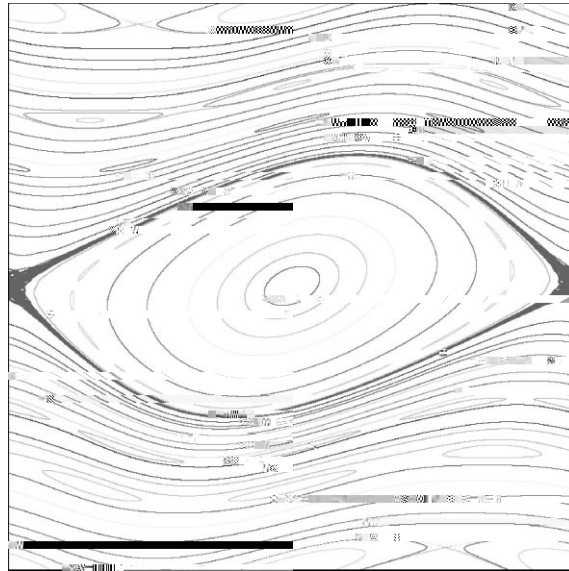


Fig. 6. Phase portrait for $k = 0.5$. The trajectory spirals inwards towards the center.

5.3. Standard map with net flux

Consider the standard map with net flux k , $(x, y) \in \mathbb{T}^2$, $(0, 2\pi) \times \mathbb{R}$. The potential function is $V(x, y) = V(2\pi y) - V(0)$.

$$x' = x + y', \quad y' = y - k(x) + \frac{1}{2}.$$

For $k < k_{cr} \approx 0.971635406$, the system has a single fixed point at $(0, 0)$. For $k = 0.5$, the system has a single fixed point at $(0, 0)$. For $k > k_{cr}$, the system has a curve of fixed points $y = 0$, $y = 2$.

$$f(x, y + 2 - m) = f(x, y) + 2(m, m). \quad (7)$$

The system is Hamiltonian with Hamiltonian function $H(x, y) = \frac{1}{2}x^2 + \frac{1}{2}y^2 - kxy + \frac{1}{4}y^4$.

$$x = \sqrt{2k}$$

The system has a curve of fixed points $y = 0$.

6. Periodic orbit

Consider the standard map with net flux k , $(x, y) \in \mathbb{T}^2$, $(0, 2\pi) \times \mathbb{R}$. The system has a periodic orbit $(x_{t-1}, x_t, t-1, t)$.

$$z_{t-1} = (x_{t-1}, x_t, t-1, t). \quad (12)$$

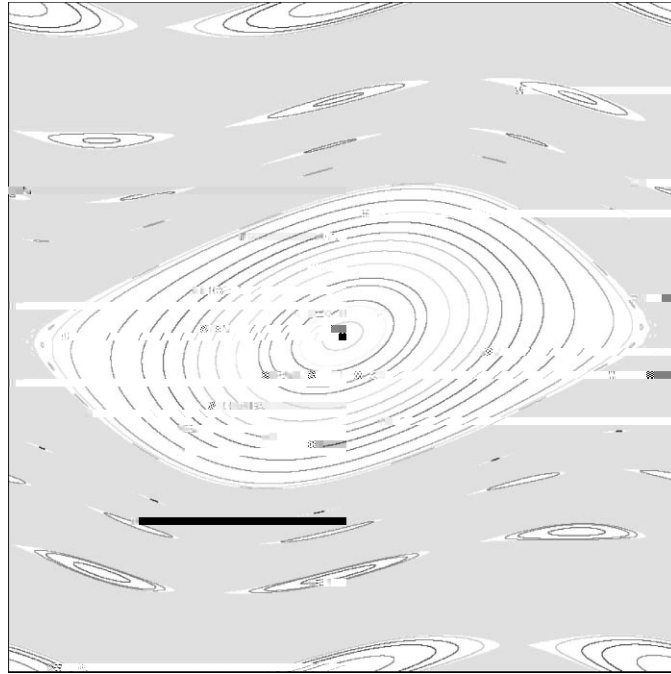


Fig. 7. Contour plot of the potential function $V(x)$ for $k = 0.5$, $F = 4^{-2}/1000$. The horizontal axis is x and the vertical axis is $V(x)$.

$$z_t = (z_{t-1}) = \begin{pmatrix} x_t \\ -x_{t-1} + 2x_t + \frac{1}{t} V(x_t) + C(t) \end{pmatrix} \quad (19)$$

where $C(t) = 0$, \dots , $x_t = c_t \in \mathcal{A}(V)$, \dots , 0 , \dots , > 0 , \dots , x , \dots

Lemma 6. Suppose that \mathcal{A} , given by (19), is a C^2 map of \mathbb{T}^4 , such that $1 + C(\cdot) \geq \epsilon > 0$. Then, for any sequence $\{c_0, c_1, \dots\}$ with $c_t \in \mathcal{A}(V) \cap \mathcal{A}$, any initial condition (z_0, z_1) , and any $\epsilon > 0$, there exists an orbit $z_t = (x_t, x_{t+1}, t, t+1)$, $t \geq 0$ of \mathcal{A} such that

$$|x_t - c_t| \leq \epsilon \quad t \geq 0,$$

provided

$$0 \leq \epsilon < \epsilon_0 = \frac{1}{(4 + a)}, \quad (20)$$

where $(a, b) \equiv \sup_{t \geq 0} |V(c_t \pm a)|$.

(1/2) $V(c_{t+1}^+)$ $> (4 + a)$ \dots a \dots $1 + C(\dots) \geq \dots$ $> 4 + a$ \dots (20).

Δ S \dots W_0 \dots $S = \mathbb{R}^2 \times (0, 1) \cap W_0$ \dots B_{t+1} \dots W_{t+1} .

z_0 S \dots T \dots S \dots U_t \dots W_t \dots $t \geq 1$ \dots B_t \dots S \dots W_0 \dots T \dots B \dots T \dots B \dots S \dots W_t .

For $|t| \leq r^t$, $r > 1$, $r^2 - wr - 1 = 0$, $w = \frac{1}{2}r(2 + |W(x)|)$.
 $|t| \leq \frac{1}{2}M^2 r^{2t}$.

For $t \leq T$, $W = 0$, $t \leq T$, $t \leq T$. \square

R a $C(\cdot)$

6.1. Standard example, continued

(15), $V(x) = k|x|$,
 $C(\cdot) = h$, $h < 1$.
 $a = 2$, 6
 $\leq 0 = \frac{k(1-h)}{4+2}$.

$M = kh$, $W = 1$, $DB)DB$, $b B Db$, $b b$, $ET bDD B$

where $h < 1$ and $\epsilon \leq 0$ 9.

7. Conclusion

... 6,9,20 ... 19 ... 16 ... (x,y) ... 17.

Appendix A

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