PHYSICA D

156 (2001) 201–218

www.elsevier.com/locate/physd

*

Department of Applied Mathematics, University of Colorado, Boulder, CO 80309-0526, USA

28 2000; 27 2001; 17 2001

Ab t act

2001

MSC: 37 40; 37 40; 37 50

Keywords:

1. I t ct

( ): $2n$ n $n > 1$, z, $z_n \to z$, U z z_n U z U. 1964 [1]

*

E-mail address: ( ).

0167-2789/01/$ 2001

0167-2789(01)00282-2

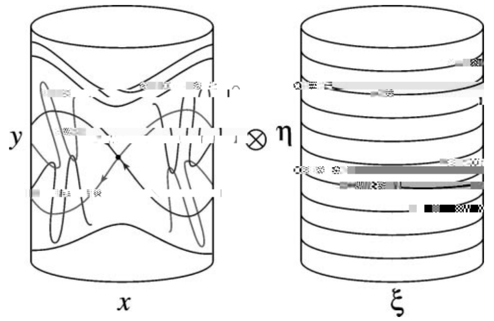


Fig. 1. [illegible] (1) [illegible] $h = 0$ [illegible] (x, y), [illegible] (ξ, η).

[illegible] $\mathrm{T}^2 \times \mathrm{R}^2$ [illegible]

$$x' = x + y', \qquad \xi' = \xi + \eta', \qquad y' = y - k(1 + h \ldots \xi) \ldots x, \qquad \eta' = \eta - kh(\ldots$$

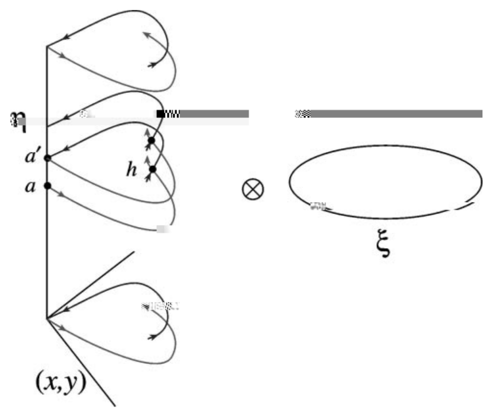


Fig. 2. [illegible] (1) [illegible] $h \neq 0$. [illegible] $\mathcal{C}$ [illegible] ξ, [illegible] C_a [illegible] C_a [illegible] $C_{a'}$ [illegible] h.

[illegible] a' [illegible] a. [illegible]

[illegible] $(x, y) \mapsto (x', y')$, [illegible]

$$x' = x + \frac{1}{\varepsilon}y', \qquad y' = y + \quad V(x). \tag{4}$$

[illegible] $n = 1$, [illegible] $V(x) = k$ [illegible] (x), [illegible] [13].

3. A t- t ab t

A[illegible] [3,4,15], [illegible] (3) [illegible]

= ∗=1,91.4887 -291.4887 - 220.1(87 -)-17,1.4887 - 1.4887 - 87 - 1 16 0 0 7.5716 177.3568 5 5256 6640

4. C[illegible] t[illegible]

[illegible]

that C is not identically zero. Then given any $a < b$, *there is a nonzero measure of initial states* (ξ_0, ξ_1) *and a sequence* $c_t \in .(V)_+ \cup .(V)_-$ *such that the solution of* (14) *has momenta,* $\eta_t = T_2(\xi_{t-1}, \xi_t)$ *satisfying* $\eta_0 < a$ *and* $\eta_T > b$ *for some time T.*

P ... $c_- \in .(V)_-$ ... $c_+ \in .(V)_+$ ... ξ_t, ... $x_t = c_\pm$ ... $(C(\xi_t)) = \pm 1$.
... (14) ...

$$\tilde{L}(\xi, \xi') = T(\xi, \xi') + W(\xi) + \tilde{C}(\xi),$$

... $\tilde{C}$ ...

$$\tilde{C} = V(c_\pm(\xi)) \quad C(\xi) \geq 0.$$

... $\tilde{C}(\xi + 2\pi) - \tilde{C}(\xi) > 0$, ... $= \tilde{L}(\xi + 2\pi, \xi' + 2\pi) - \tilde{L}(\xi, \xi')$ ... 5, ... □

4.2. Standard example

$$L(x, x', \xi, \xi') = \tfrac{1}{2}\varepsilon(x' - x)^2 + \tfrac{1}{2}(\xi' - \xi)^2 + k \;.. x(1 + h \;.. \xi), \quad (15)$$

... $k > 0$ ... $h > 0$ ... (1), ... (15) ... h

$\xi_t \to \pi$, … $\eta^* = 2\pi m$. … $(0, 2\pi m)$ … $(\pi, 2\pi m)$, … (16). … $\eta^* = (2m+1)\pi$, … $(0, \eta^*) \mapsto (\pi, \eta^*)$. … (16) … (…). … (16) …

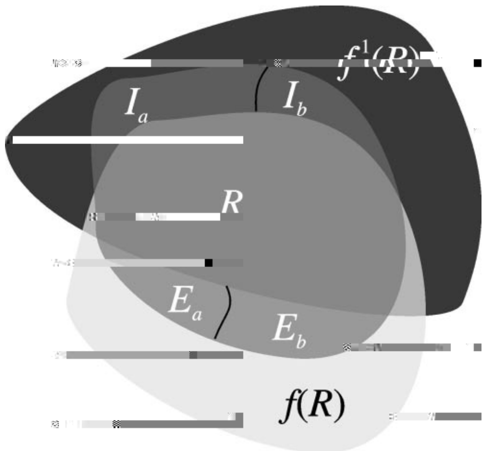
… (16), … g^* … g … g^* … g. … (16) … 2π … $\mathrm{T}^2 = \{(\xi, \eta) : 0 < \xi, \eta < 2\pi\}$. …

$$\eta^* = (\eta_{t+1} - \eta_t)^* = \lim_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} (U'(\xi_t)).$$

…

$$\langle \eta^* \rangle = \frac{1}{4\pi^2} \int_{\mathrm{T}^2} U'(\xi) = \frac{}{2\pi}.$$

… (16) … .012



. 4. … R.

$$E=\{z\in R : f(z)\notin R\}=R\setminus f^{-1}(R).$$

… f … μ, … R … :

$$\mu(E)=\mu(R\setminus f^{-1}(R))=\mu(R)-\mu(R\cap f^{-1}(R))=\mu(R)-\mu(f(R)\cap R)=\mu(R\setminus f(R))=\mu(I). \qquad (17)$$

… S^t … t. … I … *stays* … R … t. …

$$S^0=I, \qquad S^t=f(S^{t-1})\cap R=f(S^{t-1}\setminus E).$$

… S^t … $f^{-j}(S^t)$ … R … $j=0,\dots,t$, … $(t+1)^{st}$, … S^t … R, … $\sum_{t=0}^{\infty}\mu(S^t)<\mu(R)$. … $\mu(S^t)\to 0$ … $t\to\infty$. … I … R … E.

E_b

$$\mu(p(I_a)\cap E_b)=\mu(p(I_a))-\mu(p(I_a)\cap E_a)\geq \mu(I_a)-\mu(E_a). \qquad \square$$

A f_t R, t

$$I_t=R\setminus f_t(R),\qquad E_t=R\setminus f_t^{-1}(R).$$

f_t (17), R k t S_k^t.

$$S_k^k=I_{k-1},\qquad S_k^{t+1}=f_t(S_k^t\setminus E_t).$$

A S_k^t t $k\leq t$ R

$$\sum_{k=-\infty}^{t}\mu(S_k^t)<\mu(R) \tag{18}$$

t.

Lemma 4. *Let f_t be a sequence of measure-preserving homeomorphisms, and R a measurable set with incoming sets I_t and exit sets* t 1012(

5.2. Maps of the cylinder

[illegible] C, [illegible] C [illegible] $f(C)$

$$= \int_C y'\, dx' - y\, dx.$$

[illegible] C. [illegible] C, [illegible] T [illegible] B. [illegible] $U \subset T$ [illegible]

$$U = \{z \in T : f^{-1}(z) \in B\}.$$

[illegible] $D \subset B$ [illegible]:

$$D = \{z \in B : f^{-1}(z) \in T\}.$$

[illegible] $\mu(U) - \mu(D) =$ [illegible]. [illegible] 4 [illegible]

Corollary 5. *Suppose that f_t is a sequence of area and end-preserving homeomorphisms of the cylinder, and that the net flux $\Phi_t \geq \delta > 0$. Let A denote the annulus bounded by the circles $\{y = a\}$ and $\{y = b\}$ where $a < b$. Then, there is a set of positive measure of orbits that cross the annulus.*

Proof. [illegible] $U_t(a)$ [illegible] $D_t(a)$ [illegible] $\{y = a\}$ [illegible] f_t, [illegible] b, [illegible] 5. [illegible] A [illegible] $I_t = U_t(a) \cup D_t(b)$, [illegible] $E_t = f_t^{-1}(D_t(a)) \cup f_t^{-1}(U_t(b))$. [illegible] $\mu(U_t(b)) \geq \delta > 0$, [illegible] 4, [illegible] a [illegible] b [illegible] $f_t^{-1}(U_t(b))$. □

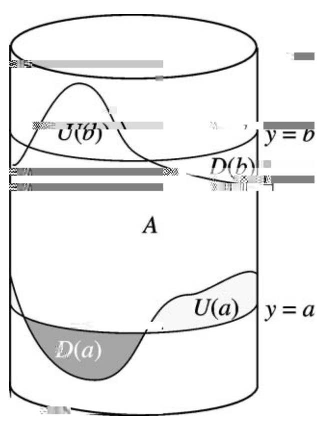
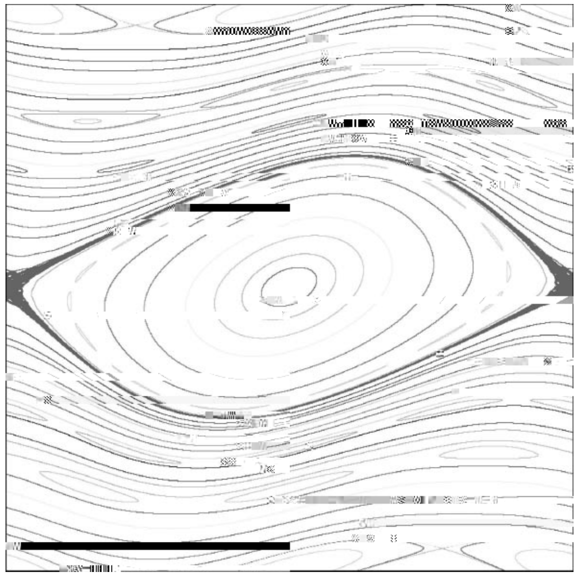


Fig. 5. [illegible]



[illegible]. 6. [illegible] $k = 0.5$. [illegible]

5.3. Standard map with net flux

[illegible] (4) [illegible] $(0, 2\pi) \times \mathbb{R}$. [illegible] $= V(2\pi) - V(0)$. [illegible] V [illegible]:

$$x' = x + y', \qquad y' = y - k \ [illegible] (x) + \frac{1}{2\pi} \ [illegible].$$

[illegible] $= 0$, [illegible] $k < k_{cr} \approx 0.971635406$ [illegible] $k = 0.5$ [illegible] 6. [illegible] 5, [illegible] $\neq 0$ [illegible] $y = 0$ [illegible] $y = 2\pi$. [illegible] m, $f(x, y + 2\pi m) = f(x, y) + 2\pi(m, m)$. [illegible] 7, [illegible] $y = 0$ [illegible]

$$x = [illegible] \overline{2\pi k}$$

[illegible] $= 2\pi k$.

6. P[illegible]t[illegible]c[illegible]t

[illegible] (12) [illegible] ε. [illegible] $z_{t-1} = (x_{t-1}, x_t, \xi_{t-1}, \xi_t)$. [illegible]

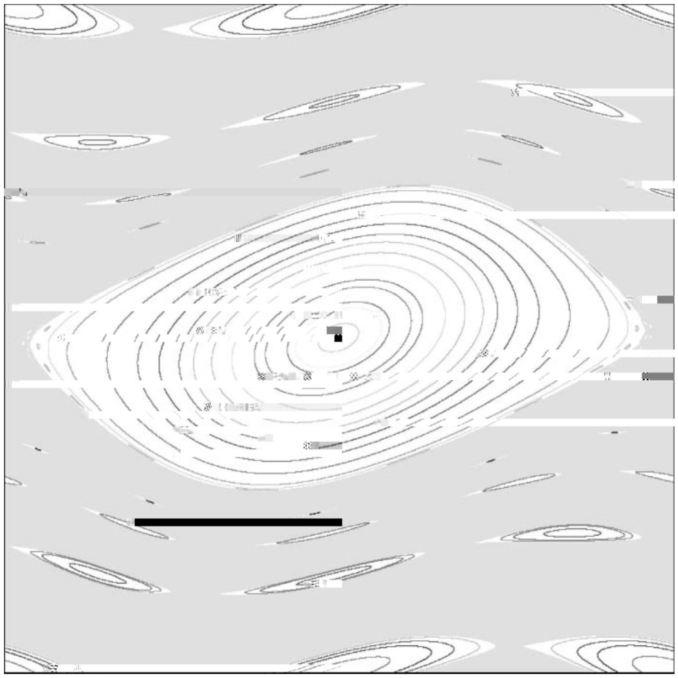


Fig. 7. … $k = 0.5$, … $\mathcal{F} = 4\pi^2/1000$. … (…).

$$z_t = \phi_\varepsilon(z_{t-1}) = \begin{pmatrix} x_t \\ -x_{t-1} + 2x_t + \frac{1}{\varepsilon} V(x_t)[1 + C(\xi_t)] \\ \xi_t \\ -\xi_{t-1} + 2\xi_t + W(\xi_t) + V(x_t) C(\xi_t) \end{pmatrix}. \tag{19}$$

… $\varepsilon = 0$, … $x_t = c_t \in$ … (V) … ϕ_0. … $\varepsilon > 0$, … x- …

L… a 6. *Suppose that ϕ_ε, given by (19), is a C^2 map of T^4, such that $1 + C(\xi) \geq \tau > 0$. Then, for any sequence $\{c_0, c_1, \dots\}$ with $c_t \in$ … $(V) \cap \mathcal{A}$, any initial condition (ξ_0, ξ_1), and any $\delta > 0$, there exists an orbit $z_t = (x_t, x_{t+1}, \xi_t, \xi_{t+1})$, $t \geq 0$ of ϕ_ε such that*

$$|x_t - c_t| \leq \delta \quad \text{…} \quad t \geq 0,$$

provided

$$0 \leq \varepsilon < \varepsilon_0 = \frac{\sigma\tau}{(4\delta + a)}, \tag{20}$$

where $\sigma(\delta, b) \equiv$ … $_{t \geq 0}|\ V(c_t \pm \delta)|$.

a $1 + C(\xi) \geq \tau$

$(1/\varepsilon)\; V(c^{+}_{t+1})\tau > (4\delta + a)$. ($\varepsilon$) $\sigma\tau/\varepsilon > 4\delta + a$ (20).

W_t B_{t+1} W_{t+1}.

S – W_0 $S = \mathrm{R}^2 \times (\xi_0, \xi_1) \cap W_0$.

z_0 S T S

U_t W_t $t \geq 1$. B

S

W_0, T B

T B W_t

S

$|\lambda_t^i| \le r^t$, $r > 1$ $r^2 - wr - 1 = 0$, $w = \;_x(2 + |\;W(x)|)$.

$$|\lambda_t| \le \tfrac{1}{2} M\delta^2 r^{2t}.$$

$W = 0$, λ_t t^2, T γ δ $|\lambda_t| \le \gamma$ $t \le T$. □

R a ξ_t $C(\xi)$,

6.1. Standard example, continued

7 (15), $V(x) = k\;\;x$, $\delta < \pi/2$, $\sigma = k\;\;\delta$. $C(\xi) = h\;\;\xi$, $\tau = 1 - h$, $h < 1$. $c_t \in \{0, \pi\}$, (x, y) $a = 2\pi$. 6

$$\varepsilon \le \varepsilon_0 = \frac{k(1-h)\;\;\delta}{4\delta + 2\pi}.$$

7 $M = kh$ $W = 1$

$h < 1$ $\varepsilon \leq \varepsilon_0$. ε 9.

7. C c

7 A 6,9,20 19 A 16 (x, y) 17.

Ac t

1999. 4. -9971760. -9810751.

R c

[1] A 5 (1964) 581 585.
[2] A 1978.
[3] A ( ), 44 A A 1992, 7 54.
[4] A 86 (1995) 284 296.
[5] 12 (2) (1999) 365 388.
[6] A A 14 (1984) 289 304.
[7] A A 2 209 (2000) 353 392.
[8] 4 (1991) 583 590.
[9] A A 2 38 A 1992.

[10] A 269 (2000) 309 318.
[11] ( ), (A , 1995), 1999, . 168 183.
[12] . 56 (1) (1996) 40 56.
[13] 64 (3) (1992) 795 848.
[14] 7 (1997) 139 147.
[15] 5 (1992) 149 160.
[16] 2000.
[17] 134 (1999) 153 184.
[18] 23 (1989) 74 76.
[19] 57, 1979, . 272 301.
[20] 56 (4) (1984) 737 754.