Ninth Annual Front Range
Applied Mathematics Student Conference
March 2, 2013

Registration: 8:30 - 9:00

Morning Session I - NC1311
9:00 - 11:35

9:00 - 9:20	Yi-Hung Kuo *University of Wyoming*	Analysis of Dispersion-Relation-Preserving Schemes
9:25 - 9:45	Jennifer Maple *Colorado State University*	Steady State Hopf Mode Interaction in Anisotropic System
9:50 - 10:10	Jason M. Gates *Colorado School of Mines*	Developing Adaptive Multiscale Finite Element Methods for Heterogeneous Media
10:10 - 10:25	15 Minute Break	
10:25 - 10:45	Victoria Li *University of Colorado, Boulder*	Mathematically Modeling Tsunamis
10:50 - 11:10	Prosper K. Torsu *University of Wyoming*	A Method of Perturbation for Solving Semilinear Elliptic Problems
11:15 - 11:35	John Villavert *University of Colorado, Boulder*	An Inviscid Regularization of Hyperbolic Conservation Laws

Morning Session II - Room NC1312
9:00 - 11:35

9:00 - 9:20	Matthew Niemerg *Colorado State University*	An Introduction to Parameter Homotopies
9:25 - 9:45	John Lepird *United States Air Force Academy*	Modeling Potential Airborne Bird Strike Countermeasures
9:50 - 10:10	Eric Hanson *Colorado State University*	Vector Bundle Based Maps and Metrics for Improving Topological Signals from Persistent Homology
10:10 - 10:25	15 Minute Break	

10:25 - 10:45	Jennifer Diemunsch *University of Colorado, Denver*	2-Factors with a Bounded Number of Odd Cycles
10:50 - 11:10	Sofya Chepushtanova *Colorado State University*	Comprehensive Analysis of Hyperspectral Data Using Band Selection Based on Sparse Support Vector Machines
11:15 - 11:35	Mark Mueller *University of Colorado, Denver*	The Physics of Extracting Coal Bed Methane

Morning Session III - Room NC1313
9:00 - 11:35

9:00 - 9:20	Ruth A. Martin *University of Colorado, Boulder*	Three-Wave Mixing and the Explosive Instability
9:25 - 9:45	Jewell Anne Hartman *University of Colorado, Colorado Springs*	Partial Differential Equations for a Statistical View of Biological Dynamics for the Interdisciplinary Studies of the Mathematical and Physical Biology of the Cell in the Advancement of Medicine
9:50 - 10:10	Quanling Deng *University of Wyoming*	Symmetric Interior Penalty Galerkin Method for Solving Semilinear Elliptic Problems
10:10 - 10:25	15 Minute Break	
10:25 - 10:45	Francis C. Motta *Colorado State University*	On Optimally Topologically Transitive Orbits
10:50 - 11:10	Bradley Lowery *University of Colorado, Denver*	Stability and Parallel Performance Analysis of Oblique *QR* Factorization Algorithms
11:15 - 11:35	Bradley McCaskill *University of Wyoming*	Using a Robin Boundary Condition within Domain Decomposition to Solve Elliptic Partial Differential Equations

Lunch: 11:45 - 12:25

Plenary Address: 12:30 - 1:30, NC1130

Dr. Loren Cobb, *University of Colorado, Denver*

Mathematics of Society and its Dysfunctions

Group Photographs at 1:30

3:00 - 3:20	Steven Ihde *Colorado State University*	Numerical Preconditioning for Homotopy Continuation
3:25 - 3:45	Timothy Morris *University of Colorado, Denver*	Cloud Conveyor System

Afternoon Session III - Room NC1313
1:45 - 3:20

1:45 - 2:05	Brandon A. Mueller *United States Air Force Academy*	Validating Falcon Telescope Network Simulations with Preliminary Data
2:10 - 2:30	Volodymyr Y. Kondratenko, Jonathan D. Beezley, Jan Mandel and Adam K. Kochanski *University of Colorado, Denver*	Assimilation of Fire Perimeter Data into the Fire Spread Model SFIRE Coupled with the WRF Model
2:35 - 2:55	Brent Davis *Colorado State University*	Application of Numerical Algebraic Geometry to Geometric Data Analysis
3:00 - 3:20	Lawrence Bush *University of Wyoming*	Conservative Flux from the Continuous Galerkin FEM
3:25 - 3:45	Ryan Hartz *United States Air Force Academy*	Testing the E ects of Altitude on Rocket Motor Performance

Plenary Speaker

12:30 - 1:30, NC1130

Mathematics of Society and its Dysfunctions

Dr. Loren Cobb
Department of Mathematical and Statistical Sciences and Department of Sociology
University of Colorado, Denver

Social problems are not the traditional domain of applied mathematics, yet remarkable progress is being made, mostly by physicists. Even most mathematicians are unaware of the impending explosion of new ideas and new mathematics for the social sciences. As recently as 25 years ago this endeavor simply did not exist. Over the past 20 years I have worked as a free-lance applied mathematician with a dozen different countries on their toughest social problems: economic (poverty, underground economies), criminal (drug trafficking, juvenile gangs), political (corruption, ethnic relations, reform of the police), and demographic (refugees, migration, population growth). In every case there has been wonderful mathematics to be discovered, and nontrivial questions of stability and change to ponder. This is a field in its infancy, where new ideas are welcomed like water for a man dying of thirst.

MORNING SESSION I

ANALYSIS OF DISPERSION-RELATION-PRESERVING SCHEMES

Yi-Hung Kuo
University of Wyoming

A numerical scheme which has the same disto adaptively resolve multiscale features (such as those described by random fields) as well as possible extensions to time-dependent problems and/or three dimensions.

MATHEMATICALLY MODELING TSUNAMIS

Victoria Li
University of Colorado, Boulder

Theoretically, tsunamis are described by a system of nonlinear hyperbolic partial differential equations (PDEs) called the Shallow Water Equations. We use the method of characteristics and shock theory to solve for the propagation of the wave numerically, given a set of initial conditions. This is part of a larger project to develop an accurate tsunami forecast model, a collaboration between Professor Segur, the National Oceanic and Atmospheric Administration (NOAA), and other researchers. In the past, the Korteweg-deVries equation successfully described shock waves called solitons, but the shock waves produced by tsunamis are different and require a new model, such as the model proposed by Serre, Su-Gardner, and Green-Naghdi. Our goal is to determine whether the method of characteristics can generate results which match actual data, and if not, how the model can be improved.

A METHOD OF PERTURBATION FOR SOLVING SEMILINEAR ELLIPTIC PROBLEMS

Prosper K. Torsu
University of Wyoming

In this talk, we present a method of perturbation for solving elliptic PDEs. The method relies on representing the elliptic coefficient as a truncated Taylor series expansion. With this in place, the approximate solution is expressed as a truncated series whose n-th order term is governed by a linear elliptic problem that depends on the lower order terms for n greater than one. The only term in the series that is still governed by a semilinear elliptic equation is the zeroth order term. Each of the governing equations is obtained by performing an "order-matching" of the differential operators and the force functions. The solution of the governing equations are obtained sequentially. We establish numerical bounds for the gradient of each solution term, which indicate the stability of the solution. Numerical experiments also indicates the convergence of the true solution. One can see the potential viability of the proposed method: we only need to solve one nonlinear zeroth order equation (with elliptic coefficient 1) and linear equations for the higher order terms (also with elliptic coefficient 1). This is especially pronounced for Monte Carlo type simulations.

AN INVISCID REGULARIZATION OF HYPERBOLIC CONSERVATION LAWS

John Villavert
University of Colorado, Boulder

This talk introduces a technique that utilizes a filtering or spatial averaging of the nonlinear terms in the hyperbolic equations as an inviscid regularization of shocks. A central motivation for the analytical study we present here is to promote a recently developed filtering technique, the observable divergence method, rather than viscous regularization, as an alternative to the simulation of shocks and turbulence for compressible flows while, on the other hand, generalizing and unifying previous mathematical and numerical analysis of the method applied to the one dimensional Burgers' and Euler equations. We will primarily address two fundamental issues on the mathematical analysis of this filtering technique. The first is on the global-in-time existence and uniqueness of classical solutions for the regularization under the more general setting of quasilinear, symmetric hyperbolic systems in higher dimensions. The second issue examines one dimensional scalar conservation laws and describes the sufficient conditions that guarantee the compactness property needed in showing the technique captures the unique physical solution of the original problem as filtering vanishes.

MORNING SESSION II

AN INTRODUCTION TO

illary equation. Of particular interest to industrial semiconductor dipping applications, for example, is the height rise due to wedges of various angles. However, the solution of the Laplace-Young capillary equation for small-angled wedges (0 < < $\overline{2}$, where is the angle of the contact against a at wall - dependent upon material and uid properties - and 2 is the wedge angle) is unbounded as it approaches the corner, and standard nite element methods fail to converge. We propose using the rst-order system LL* (FOSLL*) nite element method to determine numerical solutions for the Laplace-Young capillary equations for the small-angle wedge.

2-FACTORS WITH A BOUNDED NUMBER OF ODD CYCLES

Jennifer Diemunsch, Michael Ferrara, Samantha Gra eo, and Timothy Morris
University of Colorado, Denver

A 2-factor in a graph is a collection of disjoint cycles which span the graph. This talk considers the number of odd cycles in a 2-factor, with speci c interest in claw-free graphs. A 2-factor with no odd cycles is equivalent to a pair of disjoint perfect matchings, so we conclude with a discussion of 2-factors with no odd cycles.

COMPREHENSIVE ANALYSIS OF HYPERSPECTRAL DATA USING BAND SELECTION BASED ON

adsorption and diffusion, as well as stress-dependent mechanical properties of coal such as permeability.

MORNING SESSION III

THREE-WAVE MIXING AND THE EXPLOSIVE INSTABILITY

Ruth A. Martin
University of Colorado, Boulder

The resonant interaction of three waves is one of the simplest forms of nonlinear interaction for dispersive waves of small amplitude. This behavior arises frequently in applications ranging from nonlinear optics to internal waves through the study of the weakly nonlinear limit of a dispersive system. The slowly varying amplitudes of the three waves satisfy a set of coupled, nonlinear partial differential equations known as the three-wave equations. Furthermore, without spatial dependence, the PDEs reduce to a set of coupled ODEs that constitute a completely integrable Hamiltonian system. Solutions to the ODEs can exhibit two distinct behaviors, depending on the parameters of the problem. In the first case, solutions are bounded, while in the second case solutions blow up in finite time. The first case is referred to as the nonexplosive case, while the second case is known as the explosive instability. Solutions in the explosive case can be written in terms of a Laurent series with six free parameters.

By considering the ODEs in their Hamiltonian form, we are able to reduce the two distinct cases to a single case. The solution to the ODEs can be written in terms of elliptic functions, and the parameters of the elliptic function, along with the initial data, help determine whether we are in the explosive or the nonexplosive case. We also find that every nonexplosive case has a related explosive case, while the reverse is not true. Currently, the explosive case is better understood than the nonexplosive case. However, we can take advantage of the relationship between the two cases in order to gain information about the nonexplosive case. When spatial dependence is re-introduced, we hope to use our knowledge of the analytic structure of the ODEs, combined with the integrable structure of the PDEs, in order to solve the full three-wave equations in new configurations.

PARTIAL DIFFERENTIAL EQUATIONS FOR A STATISTICAL VIEW OF BIOLOGICAL DYNAMICS FOR THE INTERDISCIPLINARY STUDIES OF THE MATHEMATICAL AND PHYSICAL BIOLOGY OF THE CELL IN THE ADVANCEMENT OF MEDICINE

Jewell Anne Hartman
University of Colorado, Colorado Springs

An interdisciplinary frontier exists between mathematics, physics, biology, and medicine; this frontier is known as biophysics. To expand this frontier, sophisticated techniques and tools from the field of applied mathematics are necessary to benefit the field. In order to study biological dynamics, analyzing rates of biochemical processes is the first step in understanding the fundamentals of biophysics. One of the tools necessary involves the field of partial differential equations, both linear and nonlinear, and it is greatly significant to the field.

To study the statistical view of biological dynamics, it is necessary to understand diffusion. Ficks Law, the Diffusion Equation, and the Conservation of Mass will be used to study concentration fields and diffusive dynamics. Biological cells can be thought of as chemical factories, and it is a known fact that chemicals diffuse. Consequently, by studying partial differential equations, rate equations for polymerization reactions, dynamics of molecular motors, macromolecule decay, and enzyme kinetics are just a few of the fields of biophysics that can be gained from applied mathematics. In order to study these systems, applied mathematics is a necessary field. Statistical biological dynamics also involves electrostatics. Problems involving electrostatics require Maxwell Equations, which are some of the most significant differential equations that exist. Diffusion equations for biophysics require knowledge of partial differential equations in an applied sense, modeling the situation to study the specifics of the field. The ultimate goal of the UCCS BioFrontiers Institute Research Program is to expand research from an interdisciplinary program combining biology, computer science, mathematics, medicine, and physics. But, without applied mathematics, these advances could not be made. Therefore, in order to succeed in accomplishing the future of medicine as the frontier between physics and biology is discovered, applied mathematics will serve as the tool that makes it all possible.

SYMMETRIC INTERIOR PENALTY GALERKIN METHOD FOR SOLVING SEMILINEAR ELLIPTIC PROBLEMS

Quanling Deng
University of Wyoming

We consider numerical discretization of a class of semilinear elliptic problems using the Symmetric Interior Penalty Galerkin (SIPG). The resulting algebraic system is then linearized using Newton's method of iteration. Several numerical examples are presented to illustrate performance of the numerical procedure. Problems such as Bratus equation and nonlinear polynomial reaction are used as benchmarks. We will also discuss a future work on Jacobian-Free-Newton-Krylov DG methods for nonlinear PDEs.

ON OPTIMALLY TOPOLOGICALLY TRANSITIVE ORBITS

Francis C. Motta, Patrick D. Shipman, and Bethany Springer
Colorado State University

We introduce a function $E : \{\ _x | x \in M\} \to \mathbb{R} \cup \{\infty\}$ on the orbits of a discrete-time dynamical system, $\ (x) : M \to M$ defined on a compact manifold, which we interpret as a measure of an orbit's topological transitivity. Motivated by phyllotactic patterns and problems in mixing, we study the family of dynamical systems $R\ : [0,1) \to [0,1)$ ($\ \in (0,1)$) defined by $R\ (x) = (x + \ ) \bmod 1$. Utilizing a recursive formula derived from the three-distance theorem, we compute exact values of $E(\{R^t(x) | t \in \mathbb{N}\}, x \in [0,1))$, for various choices of . We then study this measure of topological transitivity on transitive orbits of Bernoulli shift maps defined on sequences over finite alphabets.

STABILITY AND PARALLEL PERFORMANCE ANALYSIS OF OBLIQUE *QR* FACTORIZATION ALGORITHMS

Bradley Lowery
University of Colorado, Denver

We study parallel *QR* factorization algorithms, where the columns of the *Q* factor are orthogonal with respect to an oblique inner product. The oblique inner product is defined by a symmetric positive definite matrix, *A*. We present new stability analysis for existing algorithms as well as a new, stable, communication avoiding algorithm. We analyze both the loss of orthogonality and representativity and develop test cases to exhibit different behavior on the bounds. The numerical experiments show the bounds tight. We find that loss of orthogonality, even for the most stable algorithms, may at worst be proportional to the condition number of *A*. However, this depends greatly on the column space of the matrix.

Finally, we consider the parallel performance of the algorithms. The computation of a matrix-vector and matrix-matrix product with *A* is the overhead when comparing the cost to *QR* factorizations schemes with Euclidean inner product. To demonstrate the two extremes, we consider the cases where *A* is dense and when *A* is sparse, specifically when *A* is tridiagonal.

USING A ROBIN BOUNDARY CONDITION WITHIN DOMAIN DECOMPOSITION TO SOLVE ELLIPTIC PARTIAL DIFFERENTIAL EQUATIONS

Bradley McCaskill
University of Wyoming

In this presentation, we develop a non-overlapping iterative domain decomposition technique to solve an elliptic boundary value problem. The interface condition that is used within the iteration is a Robin boundary condition, which would allow solving each sub-domain independently. This way, the iteration is performed only at the interface level. Each local problem is solved using some type of finite volume procedure. Several numerical experiments are discussed to illustrate the important features of the procedure.

AFTERNOON SESSION I

Modeling Contest, Problem A

COOKING THE ULTIMATE BROWNIE: INTRODUCTION TO THREE-DIMENSIONAL FINITE ELEMENT ANALYSIS

Nathan Pelc
Collaborators: Jack Lepird and Austin Howard
United States Air Force Academy

Nobody likes burnt brownies, yet common rectangular brownie pans create burnt corner brownies and undercooked center brownies. Conversely, circular pans use oven area inefficiently, and still cook the outer edges quicker. In the recent COMAP MCM competition, one of the problems required modeling the outer temperature distribution of various basic brownie pans, and optimizing a pan in order to obtain uniform temperatures and maximum amounts of brownie. We, however, extended this problem to three dimensions. Therefore, our research focused on three issues: first, to accurately model the three-dimensional temperature distributions of various arbitrarily shaped brownie pans; second, to create a brownie pan that minimizes temperature differences throughout the brownie; and finally, to maximize the number of such brownie pans that can fit into an oven. To solve the problem, we used a finite element analysis (FEA) and forward difference approximation in order to numerically solve the 3D heat equation for arbitrarily shaped pans. We also created an optimization of that model to minimize the temperature differences throughout the brownie by adding and removing brownie to locations that were too hot or too cold, respectively. Finally, we also used a system of homogeneous ordinary differential equations (ODEs) to simulate the heat flow within an oven and optimize for the maximum amount of edible brownie cooked per batch.

Modeling Contest, Problem A

THE ULTIMATE BROWNIE: A GUIDE TO MAKE THE ULTIMATE BROWNIE PAN

We generalize this model to the important case of edge weights drawn from an exponential family distribution. This *weighted stochastic block model* (WSBM) includes as special cases most standard distributional forms, and thus allows us to use weighted relations directly in recovering latent block structure. Handling these general weight distributions presents several technical di culties for model estimation, which we solve using a Bayesian approach.

Because of the large parameter space, model estimation is a non-trivial problem. We introduce a variational algorithm to e ciently approximate the model's posterior distribution in dense graphs and a scalable belief propagation algorithm for sparse graphs.

In numerical experiments on edge-weighted networks,our weighted stochastic block model substantially outperforms other techniques in correctly recovering latent block structure, including the common approach of rst applying a single threshold to all weights and then applying the classic stochastic block model. We also apply the model to a network of human microbiome sample similarities showing potential application.

This model enables the recovery of latent strucviewed as a degree sequence analogue to the classical Turan problem, \Determine the minimum integer $(H; n)$ such that every n-term graphic sequence with sum at least $(H; n)$ is potentially H-graphic." The exact value of $(H; n)$ has been determined for a number of speci c classes of graphs (including cliques, cycles, complete bigraphs and others). In this talk, we will discuss the extension of this *potential function*, $(H; n)$, where H is a (loopless) multi-graph.

NUMERICAL PRECONDITIONING FOR HOMOTOPY CONTINUATION

Steven Ihde

models. The re has a very strong e ect on the atmosphere and changes in atmospheric state due to the re take time to develop, so the existing atmospheric circulation is no longer compatible with the modi ed re. Moreover, linearized changes to the atmospheric state have no hope of establishing the properly changed circulation in a physical balance. We have recently developed a technique for re ignition from perimeter data, which goes back in time and replays an approximate re history to allow the proper atmospheric circulation patterns to develop. Here, we extend this technique to the assimilation of a re perimeter into a developed re state with an established atmospheric state. The SFIRE model uses the level set method to simulate the re spread. Our data assimilation approach takes advantage of the manipulation of the re state through level set functions, which is much easier than manipulating the re areas directly.

APPLICATION OF NUMERICAL ALGEBRAIC GEOMETRY TO GEOMETRIC DATA ANALYSIS

Daniel Bates[1], Brent Davis[1], Michael Kirby[1], Justin Marks[2], and Chris Peterson[1]
Colorado State University[1]
Air Force Institute of Technology, Wright Patterson Air Force Base[2]

For two xed vector spaces V and W of a nite-dimensional vector space over R de ne

$$\cos(\ (V;W)) = \max_{v2V;w2W} fv\ \ wjkvk = kwk = 1g:$$

Note that

$$(V;W) = \min_{v2V;w2W} f\cos^{\ 1}(v\ \ w)jkvk = kwk = 1g$$

is called the rst principal angle between subspaces V and W.

Let fV

This is accomplished using a high altitude testing site located in Colorado, and a low altitude testing site located in Louisiana and employing hobbyist rocket motors. This research is ongoing, but promising results have been made and discrepancies between the manufacture's rocket motor speci cations and measured results have been found.